THE GALVANOMETER

A SERIES OF LECTURES

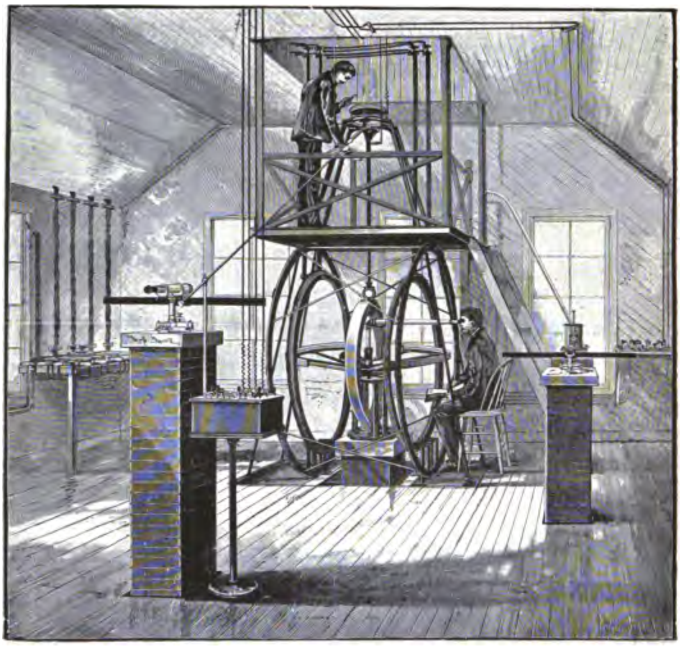
BY

EDWARD L. NICHOLS

PROFESSOR OF PHYSICS IN CORNELL UNIVERSITY

As printed in ELECTRIC POWER.

NEW YORK
McILROY & EMMET
1894



GALVANOMETER AT CORNELL UNIVERSITY.

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[illegible] NICHOLS

PROFESSOR [illegible]

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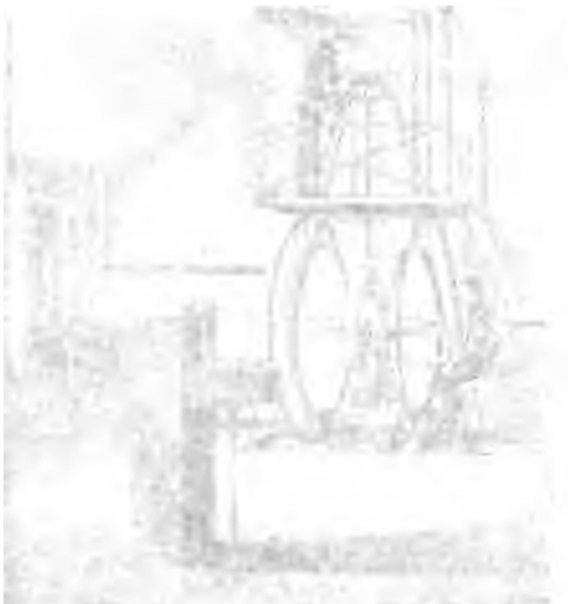

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PREFACE.

THIS series of lectures was written, in the first place, for the benefit of a class of students of electrical engineering in Cornell University. In it I have endeavored to bring together, in compact form, for the benefit of readers of limited mathematical attainments, the most important features of the theory of the galvanometer, together with some suggestions concerning the methods of using that instrument. To this end, free use has been made of the work of many writers. The treatises of Maxwell, and of Mascart and Joubert, in particular, have been repeatedly drawn upon. From various fellow physicists, also, I have received hints. The services of one of these, Professor W. S. Franklin, I desire especially to acknowledge, since several important features in the theoretical treatment of my subject are due to suggestions made by him. Detailed studies of the performance of sensitive galvanometers by Professor Ernest Merritt and Mr. F. J. Rogers, and a variety of details culled from the records, published and unpublished, of other workers in the laboratories of the department of physics have been made use of.

Nearly all that has been written about galvanometers, aside from the theory and details of construction, applies to the types of instrument which were called into existence by the requirements of the testing of cables and telegraph lines and of submarine signalling. Concerning the use of such instruments on the one hand and upon the subject of what may be termed voltmeter and ammeter work, from its relation to the practice and meth-

ods of the dynamo laboratory, abundant material is already accessible in the manuals of Kempe, Ayrton, Kittler and many other writers. To this part of the subject I have paid little attention.

The extraordinary demands upon the sensitiveness of the galvanometer made by the researches of Langley, Aegström, Julius, Rubens, Snow, Paschen and others in the domain of radiation, has, however, resulted in the development of a new class of instruments, the delicacy of which has greatly modified the art of using the galvanometer. Of these matters I have endeavored to give some account.

EDWARD L. NICHOLS.

January, 1894.

LECTURE I.

Galvanometers for absolute measurement.—Three effects of the voltaic current may be made use of for measurement; the *thermal*, utilized in electro-calorimetry, the *chemical*, which is used in voltametry, and the *magnetic*, upon which the action of the galvanometer depends. The essential parts of this instrument are a magnet needle, suspended in general with freedom of vibration

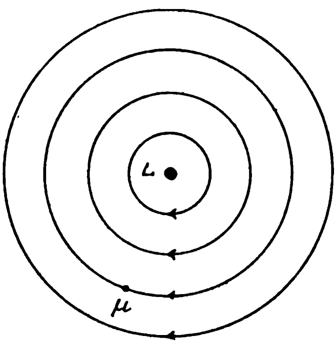

FIG. I.

about a vertical axis, and an electric circuit, consisting of one or more coils of wire, within the field of which the needle swings.

For purposes of absolute measurement the coils of the galvanometer must be of known dimensions, they must be at a known distance from the needle, and the latter must be situated in a magnetic field of known intensity. The simplest form, which is also that most frequently met with in practice, consists of one or more circular coils mounted vertically in the magnetic meridian. Where a single coil is used the needle is in its axis. When there is more than one coil, these have a common axis in which the needle hangs.

The law of action of the galvanometer may be derived from the following familiar and well-established principles:

1. *Influence of a current upon a magnetic particle.*—Given a conductor L, carrying current at right angles to the plane of the paper (Fig. 1) in the direction indicated by the lines of force. A particle μ situated in the field surrounding L will tend to move along a line of force.

2. *Influence of a circular current.*—If L is a circular conductor and μ be situated in the axis of the ring (Fig. 2) at a distance x from the plane of the latter, and at a distance d from the conductor, the action of each element $(d\,L)$ of the ring will be to tend to drive μ along

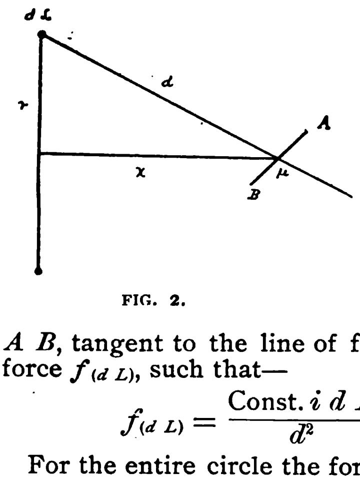

FIG. 2.

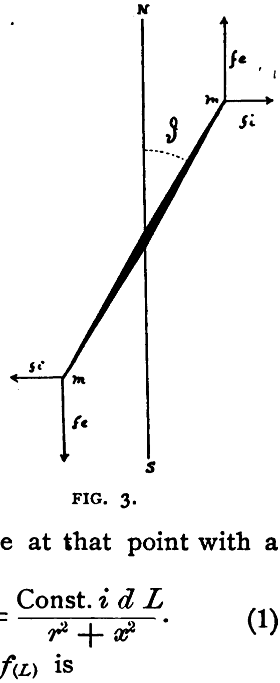

FIG. 3.

$A\,B$, tangent to the line of force at that point with a force $f_{(d\,L)}$, such that—

$$f_{(d\,L)} = \frac{\text{Const.}\, i\, d\, L}{d^2} = \frac{\text{Const.}\, i\, d\, L}{r^2 + x^2}. \tag{1}$$

For the entire circle the force $f_{(L)}$ is

$$f_{(L)} = \text{Const.}\, \frac{2\, \pi\, r\, i}{r^2 + x^2}. \tag{2}$$

If μ be a magnet-pole of strength m with freedom of motion only around a vertical axis, the case with which we have to do in considering the galvanometer, the effèctive component of $f_{(L)}$ along the axis is

$$f_i = \text{Const.}\, \frac{2 \pi r i m}{r^2 + x^2} \cdot \frac{r}{\sqrt{r^2 + x^2}} = \text{Const.}\, \frac{2\, \pi\, r^2\, i\, m}{(r^2 + x^2)^{\frac{3}{2}}}. \tag{3}$$

When r and x are taken in centimeters the constant is unity and i is expressed in absolute measure.

The actual case to be considered is that of a needle, swinging in a magnetic field. This field is made up of two active components, the horizontal force (f_e) of the earth's magnetism, or of an artificial field which takes its place, and the horizontal component of the force due to the current (f_i), see Fig. 3.

When the couple $2\,l\,f_e \sin\vartheta$, due to the action of the earth's field is balanced by the couple $2\,l\,f_i \cos\vartheta$, the needle is in equilibrium, a condition which is expressed by the equation.

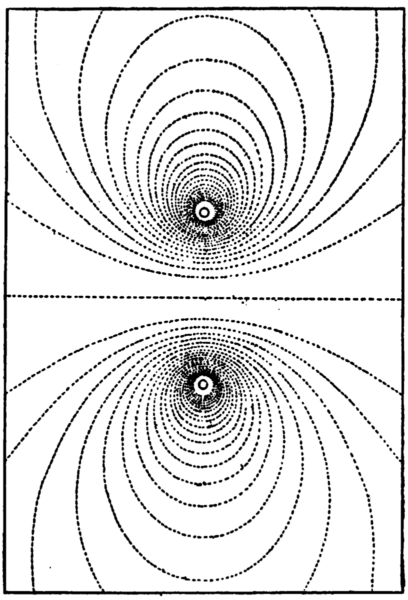

FIG. 4.

$$f_i = f_e \tan\vartheta\,; \tag{4}$$

from which, since $f_e = H\,m$, where H is the horizontal component of the earth's magnetism and m is the strength of the magnet pole, we derive the equation of the tangent galvanometer of a single turn.

$$i = \frac{(r^2 + x^2)^{\frac{3}{2}}}{2\,\pi\,r^2}\,H \tan\vartheta. \tag{5}$$

If the galvanometer consists of any number of separate coils the radii of which are r_1, r_2, r_3, r_4, etc., at dis-

tances x_1, x_2, x_3, x_4, etc., from the needle, containing respectively n_1, n_2, n_3, n_4, etc., turns of wire, each coil will have its independent action upon the needle, and the following general equation of the galvanometer may be written. This equation is applicable in every case in which the coils are in the magnetic meridian, and possess a common axis, in which axis the needle, the length of which must be small compared with the radii of the coils, is suspended. This equation is,

$$i = \frac{H \tan \vartheta}{\dfrac{2\pi r_1^2 n_1}{(r_1^2 + x_1^2)^{\frac{3}{2}}} + \dfrac{2\pi r_2^2 n_2}{(r_2^2 + x_2^2)^{\frac{3}{2}}} + \dfrac{2\pi r_3^2 n_3}{(r_3^2 + x_3^2)^{\frac{3}{2}}} + \text{etc.}} \tag{6}$$

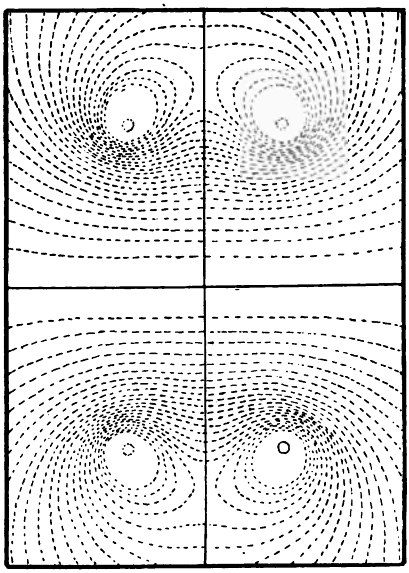

FIG. 5.

The denominator of the right-hand member of this equation is termed *the constant of the coils*, and it is designated by the letter G_1;* the ratio $\frac{H}{G}$ is *the constant* of *the galvanometer*. Equation (6), then, may be written either in the form

* The reciprocal of this expression is sometimes taken as the constant of the coils, in which case equation (7) is written

$$i = G H \tan \vartheta, \text{ instead of } i = \frac{H}{G} \tan \vartheta.$$

$$i = \frac{H}{G} \tan \vartheta. \tag{7}$$

or in the form

$$i = K \tan \vartheta. \tag{8}$$

As a matter of construction, tangent galvanometers usually have either one or two coils. The large galvanometer of Cornell University with six coils is really a combination of several instruments, designed for heavy currents and one for currents of small intensity.

Of galvanometers with a single coil, the common form has the needle in the plane of the ring. For such

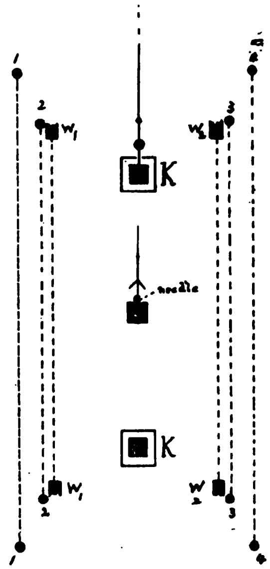

FIG. 6.

instruments the quantity x in equation (6) is zero. The equation then takes the following simpler form:

$$i = \frac{r}{2 \pi n} H \tan \vartheta. \tag{9}$$

The objection to galvanometers with the needle in the plane of the ring is in the nature of the field of force due to the current in a single ring. This field of a circular current was discussed by Lord Kelvin in 1869;* from whose results, as embodied in Plate XVIII. of the second volume of Maxwell's *Treatise on Electricity*, Fig. 4 is taken.

* W. Thomson, Transactions of the Royal Society of Edinburgh, vol. 25, p. 217.

In such a field, by the use of a long needle or the displacement of the needle from its position in the centre of the coil, considerable errors are introduced. It is to reduce such errors to a minimum that the Helmholtz form of the tangent galvanometer is used. In this instrument two coils of equal area are placed at a distance apart equal to their radius, and the needle is

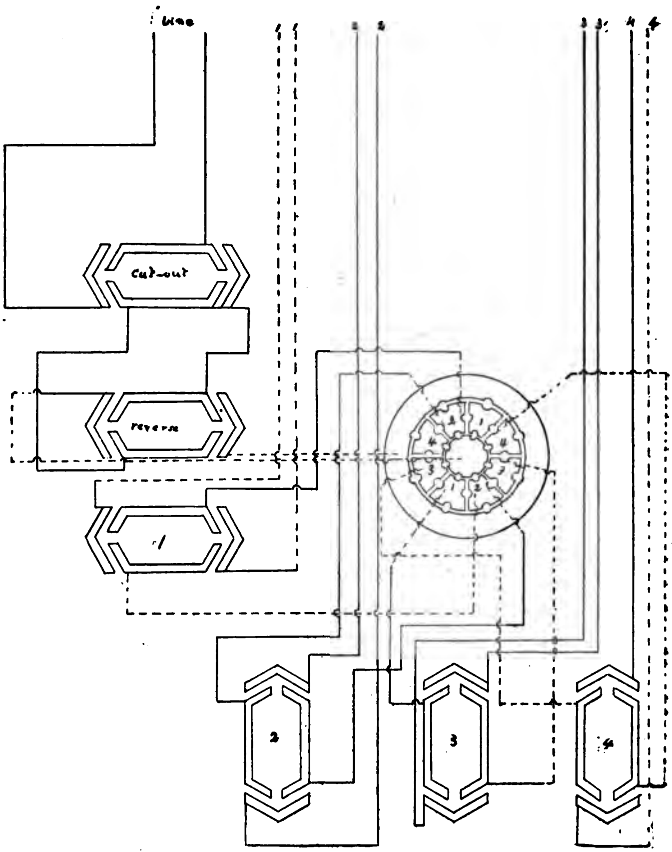

FIG. 7.

situated midway between them in their common axis. The field produced by current in coils thus located is very nearly uniform for a considerable region surrounding the centre of the system, and displacements of the needle have of but slight influence upon the performance of the galvanometer. Figure 5, also from Lord

Kelvin's paper just cited, affords a comparison of the field of two parallel circular currents, with that produced by a single coil.

The large tangent galvanometer of Cornell University, to which brief reference has just been made, affords an interesting example of the application of the Helmholtz construction. A diagram of this instrument, showing the proportions and dimensions of the various coils, is given in Fig. 6.

It consists, essentially, of four distinct instruments arranged so as to be used separately or in combination. There are:

a. A Helmholtz galvanometer (1, 4, 1, 4) with coils about two meters in diameter, each consisting of a single turn of heavy wire of about 2.00 cm. diameter.

b. A similar galvanometer symmetrically placed with reference to the first and acting upon the same needle, with coils of the same heavy wire, the diameter of each coil being about 160 cm.

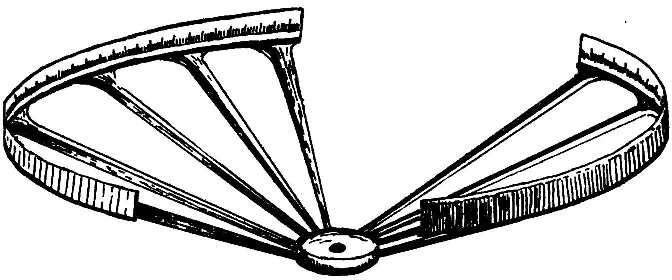

FIG. 8.

c. A Helmholtz galvanometer, for small currents, with 36 turns, divided into two sections of eighteen turns apiece, in each coil, the mean diameter of the coils about 152 cm. The centre of the system coincides with that of *a* and *b*, and the same needle serves.

d. A modification of the Kohlrausch instrument for the determination of *H.* This consists of a coil 100 cm. in diameter with 100 turns of wire. It is suspended in the magnetic meridian by means of a phosphor bronze wire about 200 cm. in length. The axis of the coil is coincident with those of the galvanometer coils already described. The method of using it will be given in the lecture on the determination of *H.*

The four coils 1, 4 and 2, 3 are connected with a switchboard of massive bronze, shown in diagram in Fig. 7, by means of which any coil can be used by itself or any two or three, or all four in series (directly or differentially) or in multiple. Thus a considerable range

of sensitiveness may be obtained. The time required to change from one combination to another is that necessary to insert and withdraw certain plugs and to throw the switches.

The following are the dimensions of the six coils and the constants of the galvanometer with various arrangements of the coils when the strength of the field is $H = 0.1710$.

Dimensions.

Measurements by Ryan and Hammon, 1886.

Coil 1	radius	= 100.1047	cm.
" 4	"	= 100 1275	"
" 2	"	= 80.1037	"
" 3	"	= 80.1025	"
" W_1	mean radius	= 76.0765	"
" W_2	" "	= 76.0919	"
Distance apart	($2x$) 1 to 4;	99.9770	"
" "	($2x$) 2 to 3;	80.0274	"
" "	($2x$) W_1 to W_2	76.0310	"

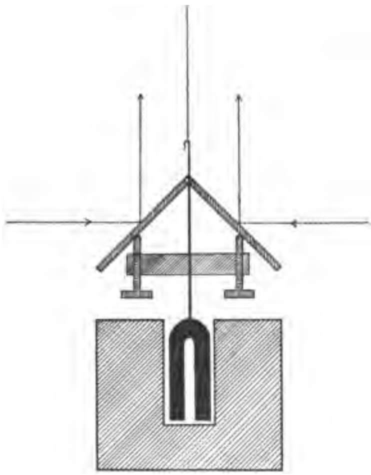

FIG. 9.

Constants (C. G. S.)

Arrangement.	G	$\log GH$	$K = \frac{H}{G}$	$\log K$
Coil 1	.044946	— 2.652690	3.8046	0.580306
" 4	.044942	— 2.652656	3.8049	0.580340
Coils 1 + 4	.089888	— 2.953703	1.9024	0.279293
Coil 2	.056159	— 2.749419	3.0449	0.483577
" 3	.056159	— 2.749421	3.0449	0.483575
Coils 2 + 3	.112319	— 1.050450	1.5225	0.182546
" 1 + 2 + 3 + 4	.202205	— 1.305792	0.84568	— 1.927204
" (1 + 4) — (2 + 3)..	.022430	— 2.350831	7.6237	0.882165
Wound coils in series....	.729752	— 1 863175	0.23433	— 1.369821

The method of reading the deflections of this galvanometer is as follows: There are two circular scales, graduated decimally upon metal strips. These strips form two opposite quadrants of the inner face of a cylinder, the radius of which is 50 cm. (see Fig. 8). At the centre of this cylinder is the mirror, circular in form but cut in two diametrically, the halves hinged at the median line and dropped down to an angle of 45° with the horizontal plane, to which position they are adjusted by means of screws (see Fig. 9).

The scale is viewed through a telescope (Fig. 10) before the objective of which is placed a large right-

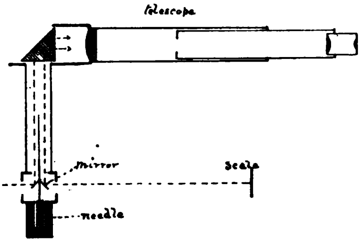

FIG. 10.

angled total-reflection prism. This catches the vertical rays proceeding from the two scales to the mirror and reflected upwards from the oblique faces of the latter. Images of those portions of both scales which are directly opposite the halves of the mirror are brought into the eye-piece in superposition. These may be read separately by cutting out one or the other by means of shutters.

This galvanometer has been briefly described by Professor W. A. Anthony,* under whose direction it was constructed in 1885.

* *The Electrical Engineer* (N. Y.), Vol. iv., October, 1885.

LECTURE II.

The sine galvanometer.—In the case of the sine galvanometer, the coils of which are free to revolve upon a vertical axis, and are made to follow the needle in its deflection, the formulæ of the tangent galvanometer are applicable with the following slight modification.

Since the deflected needle is always finally in the plane of the coils the couple due to the current acting upon it is $2\,l\,f_i$ instead of $2\,l\,f_i \cos\vartheta$, and equations 4 and 5 become

$$f^i = f_e \sin\vartheta. \tag{10}$$

$$i = \frac{H}{G} \sin\vartheta. \tag{11}$$

In the sine galvanometer the angular movement of the coils may be indicated by verniers upon an astronomical circle, a method capable of greater accuracy than the usual methods of reading the deflection of a tangent galvanometer. The limit of accuracy is determined, however, not by the fineness of the circle in question, but upon the precision with which the coincidence of the needle with the plane of the coils, in the final adjustment of the latter can be ascertained. The most refined device for this purpose is due I believe, to Professor H. A. Rowland. It consists of a small reading telescope carried upon an arm which turns with the coils of the galvanometer. This telescope bears a short horizontal scale, the central division of which is in the same vertical plane as the axis of the telescope. A mirror attached to the galvanometer needle gives an image of the scale in the eye-piece of the telescope, the zero falling upon the cross hair when the plane of the coils is parallel to the axis of the needle.

Standard galvanometers with variable constants.—It is often desirable to be able to vary the constant of a standard galvanometer in a determinate manner. To vary H in such a manner is not easily practicable, but G may be subjected to perfectly definite changes. The galvanometer of Obach* affords an illustration (Fig. 11 is from Obach's original plate) of one method of accomplishing this end. It depends upon the fact that the

* Obach: Carl's Repertorium 14, p. 507, 1878.

constant G of a galvanometer, with needle in the plane of the coil, is inversely proportional to the projection of the radius of the coil upon the vertical north and south plane. By mounting the coil of a tangent galvanometer upon a horizontal axis and causing it to make any angle θ with the vertical, the effective constant of the coil can be given any value $G_\theta = G \cos \theta$. The equation of the galvanometer then becomes

$$i = \frac{H}{G_\theta} \tan \vartheta = \frac{H}{G \cos \theta} \tan \vartheta.$$

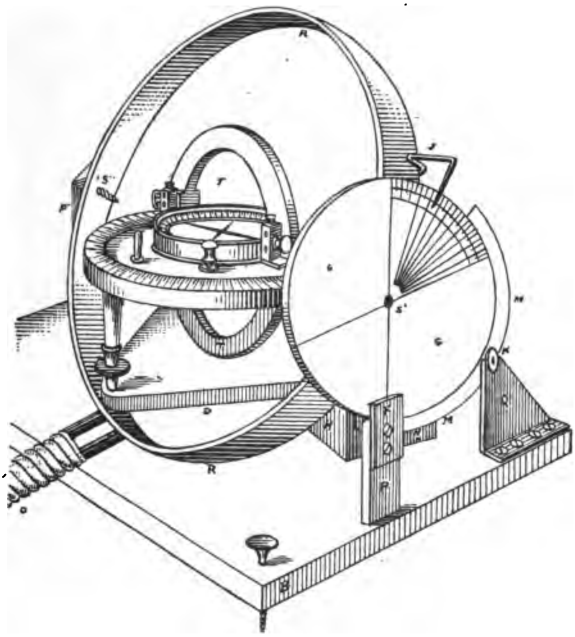

FIG. 11.

Another well-known form of standard instrument with variable constant, is the Thomson graded galvanometer, in which the needle is moved along the axis of the coil. The sensitiveness of instrument is thus within definite control through a wide range. For the double purpose of further increasing the range of usefulness and of protecting the needle from magnetic disturbance the galvanometer is provided with an artificial field. A further discussion of this feature will be given in the lecture on galvanometers with artificial fields. This

type of instrument like the "swinging-arm" galvanometer of Moler and other forms does not, however, belong to the class of galvanometers now under consideration, since the constant is determined by calibration instead of being computed directly from the dimensions of the coil and the position of the needle.

Corrections to be applied in the case of galvanometers in which the conditions already described are not fulfilled.—In the development of the law of the galvanometer: given in lecture 1, the following conditions were assumed:

1. The needle is in the axis of the coils;
2. The coils are vertical and in the magnetic meridian;
3. The length of the needle is small as compared with the radius of the coils. It is further assumed that the number of turns is so small that they can all be wound within a space such that the cross-section of the bundle of wires is small as compared with the radius of the coil.

The theory of the action of a circular current upon a needle situated at a distance from the axis is given by Maxwell† also by Mascart and Joubert,‡ and has been reproduced in many other treatises.

It will be possible here to indicate only the outlines of the analysis, and the theoretical considerations upon which the selection of certain forms, notably of the Helmholtz type of galvanometer is based.

It is usual to base this discussion upon the following considerations:

1. For any point situated at a distance y from the axis of circular magnetic layer of unit density the potential ρ is

$$\rho = 2\pi\,(f_0 + f_1\,y^2 + f_2\,y^4 + \ldots .\ \text{etc.}) \qquad (12)$$

in which f_0 f_1 f_2, etc., are all functions of the distance of the point from the plane of the sheet.

2. The potential at the same point due to a magnetic shell of unit strength, the boundary of which is the same as that in the previous case is V, where $V = -\frac{d\rho}{dx}$ whence,

$$V = -\,2\pi\left(\frac{d f_0}{d x} + \frac{d f_1}{d x}\,y^2 + \frac{d f_2}{d x}\,y^4 + \ldots\right) \qquad (13)$$

3. The x component of the magnetic force due to a circular current traversing the boundary of the magnetic shell is

† Maxwell: Electricity and Magnetism, Vol. II., Chap. 14, also in Art. 711, p. 355 (Ed., 1892).

‡ Mascart et Joubert: Lecons sur l'electricite et sur le magnetisme, T. 2, pp. 101, etc.

$$X = -\frac{d\,V}{d\,x} = 2\,\pi\left(\frac{d^2 f_0}{d\,x^2} + \frac{d^2 f_1}{d\,x^2}\,y^2 + \frac{d^2 f_2}{d\,x^2}\,y\,\ldots\right) \tag{14}$$

The quantities f_0, f_1, f_2, etc., are related to each other as follows:

$$f_1 = -\frac{1}{2^2}\cdot\frac{d^2 f_0}{d\,x^2}$$

$$f_2 = -\frac{1}{4^2}\cdot\frac{d^2 f_1}{d\,x^2} = +\frac{1}{(2\,.\,4)^2}\cdot\frac{d^4 f_0}{d\,x^4} \tag{15}$$

. . . .

$$f_n = -\frac{1}{2\,n^2}\cdot\frac{d^2 f_{n-1}}{d\,x^2} = \pm\frac{1}{(2\,.\,4\ldots n)^2}\,\frac{d_n f_0}{d\,x_n}, \text{ etc.,}$$

etc.

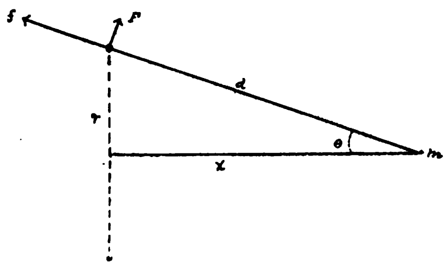

FIG. 12.

The value of f_0, however, from which all the higher coefficients are readily derived, is determined by the value of the potential upon the axis. At the distance x from the plane of the layer for example the potential is—

$$\rho_0 = 2\,\pi\,(\sqrt{r^2 + x^2} - x)\,; \tag{16}$$

where r is the radius of the circular layer, a form which leads by successive differentiation to the same expression for the magnetic action of the current as that given in lecture I. (equation 3). Thus:

$$-\frac{d\,\rho_0}{d\,x} = V_0 = -\,2\,\pi\,\frac{x}{\sqrt{x^2 + r^2}}\,; \tag{17}$$

$$-\frac{d\,V_0}{d\,x} = f_i = 2\,\pi\,\frac{r^2}{(x^2 + r^2)^{\frac{3}{2}}} \tag{18}$$

which is identical with equation 3 when the current is unity.

Since, however, ρ_0 is a particular value of ρ (equation 12), in which $y = 0$ we have

$$f_0 = \sqrt{r^2 + x^2} - x, = u - x\,; \tag{19}$$

where $u^2 = r^2 + x^2$.

The coefficients belonging to the series for X may readily be obtained by differentiation.

Thus we have

$$\begin{array}{l|l} f_0 = u - x, & \dfrac{d_2\, f_0}{d\, x^2} = -\, 2^2\ f_1 = \dfrac{d^2\, u}{d\, x^2} \\ f_1 = -\dfrac{1}{2^2}\dfrac{d^2\, u}{d\, x^2} & \dfrac{d^2\, f_2}{d\, x^2} = -\, 4^2\ f_2 = -\dfrac{1}{2^2}\dfrac{d^4\, u}{d\, x^4} \\ f_2 = \dfrac{1}{(2\,.\,4)^2}\dfrac{d^4\, u}{d\, x^4} & \dfrac{d^2\, f_2}{d\, x^2} = -\, 6^2\ f_3, \text{ etc.}\,; \end{array} \tag{20}$$

$$\frac{d\,u}{d\,x} = \frac{x}{u}$$

$$\frac{d^2\,u}{d\,x^2} = \frac{r^2}{u^2}$$

$$\frac{d^3\,u}{d\,x^3} = -\,\frac{3\,r^2\,x}{u^5}$$

$$\frac{d^4\,u}{d\,x^4} = \frac{3\,r^2\,(4\,x^2 - r^2)}{u^7}.$$

By means of these values we may write the series for X to the fourth power of x.

$$X = 2\,\pi\frac{r^2}{u^3}\left[1 - \frac{3}{2^2}\cdot\frac{4\,x^2 - r^2}{u^2}\,\frac{y^2}{u^2} + \frac{3^2\,.\,5}{(2\,.\,4)^2}\,\frac{r^4 - 12\,r^2\,x^2 + 8\,x^4}{u^4}\,\frac{y^4}{u^4}\right] \tag{22}$$

The series, the lower members of which are given in 22 leads to a result only when $y < u$, which is always the case in dealing with galvanometers of ordinary form.

When the needle lies in the axis, y becomes zero, and equation 22 becomes

$$X = f_i = \frac{2\,\pi\,r^2}{u^3} = \frac{2\,\pi\,r^2}{(r^2 + x^2)^{\frac{3}{2}}}\,;$$

which is identical, of course, with 18 and with 3.

Professor W. S. Franklin has suggested to the writer that an expression for X corresponding to 22 may be obtained without recourse to the artificial conception of the potential of a magnetic shell.

His method is as follows:

To find a development of the *component in the direction of the axis* of the magnetic field due to a circular coil.

The value of this component at points in the axis (at these points the component being the total field) is obtained as follows:

Let a magnetic pole of strength m be placed in the axis of the coil at a distance x from its plane. The magnetic field f at the wire is $f = \frac{m}{d^2} = \frac{m}{r^2 + x^2}$. An element $d\,l$ of the coil will be acted upon by a force $d\,F$ at right angles to f and to $d\,l$, such that $d\,F = f\,i\,d\,l$. i being the strength of the current in the coil. The component of $d\,F$ in the axial direction is $f\,i\,d\,l\,\text{Sin}\,\theta$, and this same force acting upon every element $d\,l$ of the coil gives for the *total* force acting on the coil (which force is in the axial direction from symmetry) $F = f\,i\,l\sin\theta$.

$$\text{where } l = 2\,\pi\,r \text{ and } \sin\theta = \frac{r}{\sqrt{x^2 + r^2}}.$$

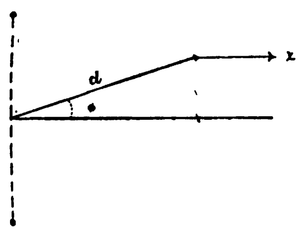

FIG. 13.

Substituting in this value for F the values of $f = \frac{m}{r^2 + x^2}$, l, and $\cos\theta$ we have

$$F = \frac{m}{r^2 + x^2}\,i\,2\,\pi\,a\,\frac{r}{\sqrt{x^2 + r^2}} = 2\,\pi\,m\,i\,a\,\frac{r}{(r^2 + x^2)^{\frac{3}{2}}} \tag{23}$$

$$= 2\,\pi\,m\,i\,\frac{r^2}{(r^2 + x^2)^{\frac{3}{2}}}.$$

Owing to the equality of action and reaction this same force must act upon the pole m, but a force acting on a magnetic pole is always the product of the strength of the pole into the *strength of the magnetic field at the pole* so that the factor by which m is multiplied in (1) is the required strength X of the magnetic field due to the

circular coil at a distance from x the plane of the coil and in its axis so that

$$X = 2\pi i \frac{r^2}{(r^2 + x^2)^{\frac{3}{2}}}$$

$$= \frac{2\pi i}{r} \frac{1}{\left(1 + \frac{x^2}{r^2}\right)^{\frac{3}{2}}},$$

in which expression $\frac{1}{\left(1 + \frac{x^2}{r^2}\right)^{\frac{3}{2}}}$ may easily be expanded in powers of $\frac{x^2}{r^2}$, giving a series for X with definite coefficients.

The *component* in any given direction of a magnetic field satisfies La Place's equation so that we may expand such a component when symmetrical to an axis in a series of zonal harmonics, *i.e.*,

$$X = A_0 P_0 (\cos\varphi) + A_1 d P_2 (\cos\varphi) + A_2 d_2 P_2 (\cos\varphi) \ldots (25)$$

where the A's are undetermined coefficients X is the required *component* at *distance* d from centre of circular coil d making an angle φ with the axis of the coil (see Fig. 13). When $\varphi = 0$ then (25) must reduce to the value (24) for X at points in the axis. Substituting in (24) the expansion of $\frac{1}{\left(1 + \frac{x^2}{r^2}\right)^{\frac{3}{2}}}$ and placing this series equal to (25) with $\varphi = 0$ we have (since $P_n (\cos\varphi) = 1$ when $\varphi = 0$ and d becomes x when $\varphi = 0$), by placing the various coefficients of x, x^2, x^3, etc., equal each to each a means for determining the values of the A's so that (25) is then the completely determined.

The series for X is made up thus:

$$\begin{aligned} P_0 (\cos\varphi) &= 1 \\ P_1 (\cos\varphi) &= \cos\varphi \\ P_2 (\cos\varphi) &= \tfrac{1}{2} (3 \cos^2\varphi - 1) \\ P_3 (\cos\varphi) &= \tfrac{1}{2} (5 \cos^3\varphi - 3 \cos\varphi), \text{ etc.} \end{aligned} \qquad (26)$$

The development thus formed holds only when $x < r$. For values of $x > r$ must be broken up so as to introduce the development of $\frac{1}{\left(\frac{r^2}{x^2} + 1\right)^{\frac{3}{2}}}$ when we may proceed as before.

The other rectangular components of the magnetic field due to a circular coil cannot be developed in a series of zonal spherical harmonics.

The value of the series (22) lies in the fact that by means of it we may discover the influence of deviations of the galvanometer needle from the axis when the needle is situated at various distances from the plane of the coils. This has been done by a graphical method in Mascart and Joubert's treatise (Vol. II., p. 103). Curves showing the sign and value of the two members containing y^2 and y^4 respectively, show that the former which is the more important becomes zero when $x = \frac{1}{2} r$ while the member containing y^4 is very small for that value of x. It is in accordance with the results of this analysis, therefore, that in the Gaugain galvanometer and in Helmholtz's pattern also the distance from the needle to the plane of the coils is always one half the radius of the latter.

It is not always allowable in computing the constant of a galvanometer, to take a mean radius r as applicable to all the turns which the coil contains. Particularly in the case of galvanometers with many turns of the wire it becomes necessary to consider the cross-section of the coil. For the general discussion of this case the reader is referred to Maxwell's treatise.*

Correction for the length of the needle.—The influence of the length of the needle is a subject which scarcely needs attention when dealing with modern galvanometers of the type now under consideration. The correction is a small one in all ordinary cases. It is a minimum in the Helmholtz galvanometer in which the distance from the plane of the coil to the needle is $\frac{1}{2} r$. In instruments of this type the correction is as follows, where length of needle is $2\,l$.

$$2\,l = .2\,r;\ \text{correction}\ .001$$

$$2\,l = .166\,r;\quad \text{“}\quad .0005,\ \text{etc.}$$

These lengths are much greater than any in use in modern tangent galvanometers, in the case of which instruments the correction becomes entirely neglible.

Correction for torsion.—In tangent galvanometers the correction for the torsion of the suspension fibre is so small that the following simple approximate method of determining it is entirely adequate.

To estimate the correction for torsion we twist the

* Maxwell: A Treatise on Electricity and Magnetism, Vol. II., Chap,. 15 p. 354 (edition, 1892).

upper end of the suspension fibre through an angle θ. and note the movement of the needle resulting therefrom. Let the angle be β; then

$$\varepsilon = \frac{\beta}{\theta}, \tag{27}$$

is the approximate factor.

Case in which the coils of the galvanometers are not in the magnetic meridian.—Let the coils make an angle α with the meridian. Then upon reversing the direction of the current through the instrument we get equilibrium for the following positions of the needle.

$$i \cos(\vartheta + \alpha) = \frac{H}{G} \sin \vartheta. \tag{28}$$

$$i \cos(\vartheta^1 - \alpha) = \frac{H}{G} \sin \vartheta^1. \tag{29}$$

Adding these equations we have

$$i\,[\cos\vartheta\cos\alpha - \sin\vartheta\sin\alpha + \cos\vartheta'\cos\alpha + \sin\vartheta'\sin\alpha = H(\sin\vartheta + \sin\vartheta') \tag{30}$$

from which by the use of the usual conversion formulæ

$$i\left[\cos\alpha\cos\frac{\vartheta - \vartheta'}{2} + \sin\alpha\frac{\sin\vartheta - \vartheta'}{2}\right] = \frac{H}{G}\cdot\tan\frac{\vartheta + \vartheta'}{2}\cdot\cos\frac{\vartheta - \vartheta'}{2} \tag{31}$$

When α is very small, the usual case, the member $\sin\alpha\dfrac{\sin\vartheta - \vartheta'}{2}$ which is the product of two small quantities disappears and $\cos\alpha$ is nearly unit. We may use then as an approximate form

$$i = \frac{H}{G}\cdot\tan\frac{\vartheta + \vartheta'}{2}. \tag{32}$$

When α is not small the complete expression must be obtained as follows:

From 28 and 29 we have,

$$\cos\vartheta\cos\alpha - \sin\vartheta\sin\alpha = \frac{H}{G}\cdot\frac{\sin\vartheta}{i} \text{ and}$$

$$\cos\vartheta'\cos\alpha + \sin\vartheta'\sin\alpha = \frac{H}{G}\cdot\frac{\sin\vartheta'}{i}\text{; also}$$

$$\cos\alpha = \frac{2\,H \sin\vartheta \sin\vartheta'}{i\,G \sin(\vartheta + \vartheta')};$$

$$\sin\alpha = \pm\sqrt{1 - \frac{4\,H^2 \sin^2\vartheta \sin^2\vartheta'}{i^2\,G^2 \sin^2(\vartheta + \vartheta')}}.$$

$$\cos\vartheta \left\{\frac{2\,H \sin\vartheta \sin\vartheta'}{i\,G \sin(\vartheta + \vartheta')}\right\} -$$

$$\sin\vartheta \sqrt{1 - \frac{4\,H^2 \sin\vartheta \sin^2\vartheta'}{i^2\,G^2 \sin^2(\vartheta + \vartheta')}} = \frac{H \sin\vartheta}{i\,G}. \tag{33}$$

When solved for i^2 this becomes

$$i^2 = \frac{H^2 \{\sin\vartheta' \cos\vartheta - \sin\vartheta \cos\vartheta')^2 + \sin^2\vartheta \sin^2\vartheta'\}}{G^2 (\sin\vartheta \cos\vartheta' + \cos\vartheta \sin\vartheta)^2}$$

which may be written in the following more convenient form

$$i^2 = \frac{H^2 (\tan\vartheta' - \tan\vartheta)^2 + 4 \tan^2\vartheta \tan^2\vartheta'}{G^2 (\tan\vartheta + \tan\vartheta')^2} \tag{34}$$

Since in nearly all cases the galvanometer is placed approximately with its coils in the magnetic meridian, the simpler expression (32) is of sufficient accuracy.

LECTURE III.

Ballistic Methods.—It is frequently desirable to determine current values by means of a single "throw" or "kick" of the galvanometer needle, instead of by the method of permanent deflections. In the measurement of induced currents, or of transient or rapidly fluctuating current of any kind this method is almost the only one.

A galvanometer thus used, an instrument, that is to say, which is arranged for measuring the time-integral Q or total charge transmitted during the flow of an electric current of short duration, is called a *ballistic galvanometer*, but the term should be applied to the method rather than to the instrument since any galvanometer may be used ballistically. It is true, however, that the method demands certain qualities which are not essential nor desirable in instruments intended for use in the method of permanent deflections. Thus there has arisen a type of galvanometer especially designed for the method of the first throw.

An expression for the performance of a galvanometer when used ballistically may be derived in the following manner. The force (f_i) (see Lecture I.) due to a current i flowing through the galvanometer coils, the constant of which is G, is

$$f_i = G\,i\,m, \tag{35}$$

and the moment of the couple at the needle pole is

$$G\,i\,2\,l\,m = G\,i\,M.$$

This couple acting during the time t, upon a needle the moment of inertia of which is I, will produce an angular velocity

$$\omega = \frac{G\,i\,m\,t}{I}. \tag{36}$$

In the case of galvanometers used ballistically, it is not with the current itself but with the time-integral ($Q = \int i\, d\, t$) that we are concerned, and equation (36) becomes

$$\omega = \frac{G\, M\, Q}{I} \tag{37}$$

From this equation, G, M and I being known, Q could be obtained; provided that the angular velocity could be directly observed.

A slightly different statement of this point is the following:

A current i in the coils of the galvanometer, which is supposed to be placed in the earth's field or in an artificial field the strength of which is H, the plane of the coils being in the magnetic meridian, produces a field perpendicular to H. The strength of this field at the centre of the coil is

$$F = G\, i \tag{38}$$

where G, as in previous equations, is the constant of the coils. Let ω be the angular velocity of the suspended parts of the galvanometer at a given instant of time. If any torque $\mathfrak{T}$ act upon the moving system, ω will change in such a manner that

$$\mathfrak{T} = I \frac{d\, \omega}{d\, t} \tag{39}$$

Now the field F, being perpendicular to the axis of the needle exerts upon it a torque $\mathfrak{T}$, which is equal to $F\, M$, or to $G\, i\, M$ (38). Since, however, $i = \frac{d\, Q}{d\, t}$, we have

$$I \frac{d\, \omega}{d\, t} = G\, M \frac{d\, Q}{d\, t} \tag{40}$$

If ω is zero at the instant of closing the galvanometer circuit, equation (40) becomes

$$I\, \omega = G\, M\, Q, \tag{41}$$

which is the same as (37), the fundamental equation of the ballistic galvanometer.

In order to render the ballistic method feasible, it is necessary to avoid the determination of the moment of inertia (I) and the magnetic moment (M) of the needle, also the impracticable operation of observing the angu-

lar velocity (ω); and to substitute the easily measured time of vibration T and the observation of the amplitude of the first throw (ϑ.) That it is possible to do so will appear at once from the consideration of the work done upon the needle, to deflect it through the angle ϑ of the first throw. To move the north-pointing pole N (Fig. 14) the strength of which is m, from a_1 to b, and the south-pointing pole S, of like strength, through a similar path, requires work against the force $H m$. This amounts to

$$2\,H\,m \times \overline{a_1\,a_2} = 2\,H\,M\,(1 - \cos\vartheta)$$

where M is the magnetic moment of the needle.

Expressed in terms of the moment of inertia (I) and the angular velocity (ω) of the returning needle, at the instant when the angle ϑ becomes zero, the equivalent

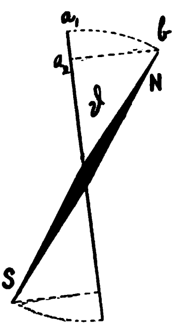

FIG. 14.

of this work is $\frac{1}{2}\,I\,\omega^2$, whence

$$\omega = 2\sqrt{\frac{H\,M}{I}}\,.\,\sin\frac{1}{2}\,\vartheta \qquad (42)$$

By combining equations 37 and 42 we obtain

$$Q = \frac{2}{G}\sqrt{\frac{H\,I}{M}}\,.\sin\frac{1}{2}\,\vartheta = \frac{2\,H}{G}\sqrt{\frac{I}{H\,M}}\,.\sin\frac{1}{2}\vartheta, \qquad (43)$$

and by use of the expression for the time of vibration

of the needle, $$T = \pi\sqrt{\frac{I}{H\,M}}, \qquad (44)$$

we may reduce the equation of the ballistic galvanometer to the final form,

$$Q = \frac{2}{\pi\,G} : H\,T\sin\frac{1}{2}\,\vartheta\,. \qquad (45)$$

Absolute measurements with the ballistic galvanometer involve :—

1. A knowledge of the constant G, to be determined by measurement.
2. A knowledge of H to be obtained by one of the methods to be described in Lecture IV.
3. A knowledge of T in seconds.
4. The observation of the throw ϑ.

The value of T is that for infinitesimal amplitudes, which is to be obtained by the application of a correction similar to that used in determining the rate of vibration of a magnetometer needle. (See Lecture IV.)

The separate determination of H and G, which latter quantity can not be satisfactorily ascertained from measurements of the coils of the forms of galvanometer usually employed in ballistic work, may be avoided by calibration of the instrument. If a known current i_p be sent through the galvanometer and the permanent deflection φ be noted, we have

$$\frac{H}{G} = \frac{i_p}{\tan \varphi}, \tag{46}$$

and the expression for Q becomes

$$Q = \frac{2\, i_p}{\pi \tan \varphi} \cdot T \sin \frac{1}{2} \vartheta . \tag{47}$$

Equation (47) affords an expression for Q in which all the factors are either numerical or capable of computation from direct observation. If the value of Q is to be free from errors other than those involved in the determination of i_p, T, φ, and ϑ however, the conditions implied in the equations upon which the expression (47) is based must be complied with. These conditions may be stated as follows:

Conditions implied by equation 41 :—

1. The needle must be at rest when the discharge through the coils begins.
2. The coils must be in the plane of the directing field (H.)
3. The field (F) due to the current in the coils must remain sensibly perpendicular to the axis of the needle during the entire discharge, so that the torque may be equal to $G\, i\, M$ through the whole time of its action. This condition compels a slow vibration period of the needle excepting in cases in which the duration of the discharge is very brief.

Conditions implied by equation 42 :—

1. The whole of the kinetic energy ($\frac{1}{2} I \omega^2$) must be employed in turning the needle against the directing field. This means that there must be no damping, a condition which can not be fulfilled in practice, and for the failure to fulfil which proper corrections must be applied.

Conditions implied by equation 44 :—

1. The suspending fibre must be free from torsion.

FIG. 15.

2. The damping must be so slight as to produce no sensible influence upon the time of vibration of the needle.

Condition implied by equation 46 :—

1. The diameter of the coils must be large compared

with the length of the needle; in other words, the law of the tangent galvanometer must hold true for such deflections as are produced by the current used in calibrating the instrument.

Many of these conditions coincide with those desirable in the magnetometer and accordingly the earlier forms of ballistic galvanometer, of which Fig. 15 affords an illustration, were simply magnetometer bars with one or more elongated coils of wire surrounding them. The requirements of modern practice and the extension of the ballistic method to operations of the highest delicacy have essentially modified the type.

The equation of the ballistic galvanometer already given (Equation 37) involves the throw of an undamped needle. In the case of a needle moving through a resisting medium (damped by friction) or in the neighborhood of metallic bodies within which induced currents are generated by the moving field of the needle (damped by induction) the deflection ϑ will always be less than that which would be observed in the case of an undamped needle, by an amount which may be estimated as follows:

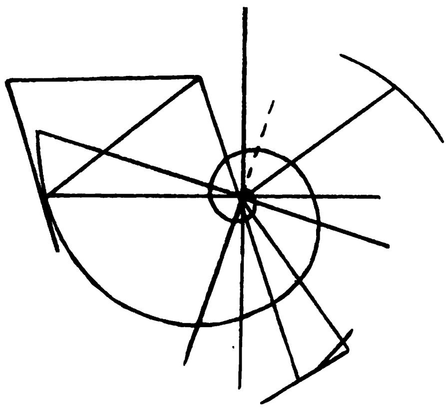

FIG. 16.

Determination of the effect of damping:—Damping affects the values of T and of ϑ. To determine its influence on these quantities we take advantage of the following law:

Law of logarithmic decrements:—The amplitudes of successive oscillations form a decreasing geometric series and the natural logarithms of successive swings have a constant difference. This difference is known as the logarithmic decrement.

The relation of the damped to undamped vibrations of the same period may be shown graphically by deriving the curve for the former by a method analogous to that in which the curve of sines is derived.

Tait* has shown that the logarithmic spiral (Fig 16) gives the law of damped vibrations in the same way that the circle leads to the curve of sines. We derive

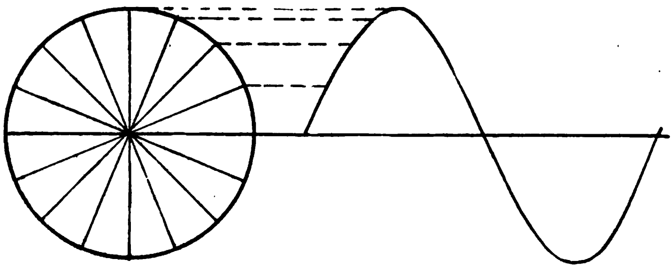

FIG. 17.

the latter curve by considering the motion of a radius of the circle traveling with a uniform angular velocity and plotting a curve (Fig. 17) with times as abscissae and the corresponding vertical distances of the moving end of the radius as ordinates. A similar procedure applied to the case of the radius vector of the spiral (Fig. 18) gives us the curve of damped vibrations.

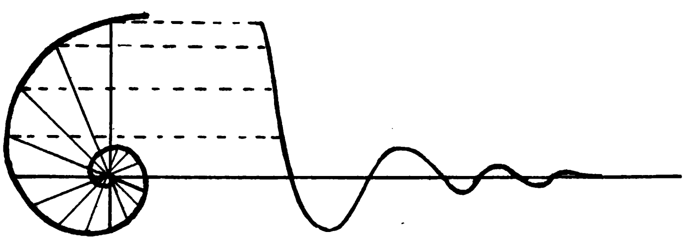

FIG. 18.

In order that the decrement shall be strictly logarithmic the resistance offered to the needle must be proportional to the velocity. Atmospheric resistance is thus proportional for low velocities and for high ones it deviates from that law and becomes more and more nearly proportional to the square of the velocity. For

*Tait; Proceedings of the Royal Society of Edinburgh, 1867; also Maxwell's Treatise, vol. ii., p. 375.

all velocities reached by galvanometer needles, however, the former law may be assumed. Even in the case of instruments of the D'Arsonval type the divergence is not marked. Fig. 19 is from the photographic trace of a very small light mirror and needle mounted in a strong field for the purpose of following the changes in rapidly fluctuating currents. [Messrs. Hotchkiss and

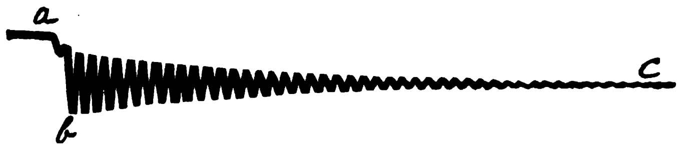

FIG. 19.

Millis, 1893.] It shows the sudden throw a_1 b_1 upon making circuit and the decrement of the oscillations as the needle comes gradually to rest in the new position at *c*. The period was about 1000 vibrations per second.

The record was obtained by shooting a sensitive photographic plate rapidly across the field of a camera and

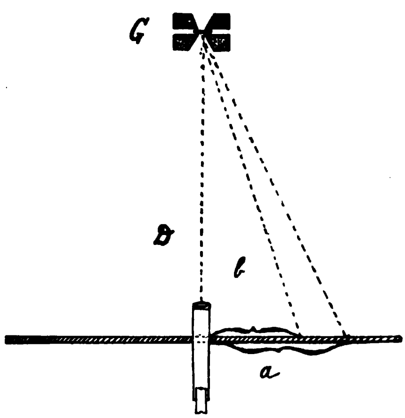

FIG. 20.

developing the image of the spot of light thrown upon the same from the vibrating mirror.

Working formulæ and approximate correction for damping:— For work requiring no very great accuracy equation (47) may be greatly simplified. Let *a* (Fig. 20) be the observed throw, in scale divisions, produced by the dis-

charge. Let b be the permanent deflection, in scale divisions produced by the current i_p and let D be the distance of the scale from the mirror. Then we have approximately Sin $\frac{1}{2}\vartheta = \frac{a}{4D}$ and tan $\varphi = \frac{b}{2D}$ whence Equation (47) becomes.

$$Q = \frac{i_p T a}{\pi b} \tag{48}$$

When a vibrating body has reached its extreme position on either side of its position of equilibrium it is said to *in elongation.*

The *zero point* (using telescope and scale) is the scale reading when the body is in its position of equilibrium. The zero point is most easily determined by observing an odd number of successive elongations, an even number to the right and an odd number to the left. The mean of the right hand readings and the mean of the left hand readings are taken. The mean of these two gives the zero point. When the damping is great this gives a distinctly erroneous value for the zero point. In such a case it is better to allow the vibrations to cease so that the zero point may be read off directly.

The distance (or angle) from the position of equilibrium to an elongation is called also an *elongation.* No confusion need arise from this double use of the word elongation.

The successive elongations of a damped vibrating body as has already been shown, form ordinarily a decreasing geometric series, in which if k be the ratio of an elongation to the next following,

$$k = \sqrt[m]{\frac{a_0}{a_m}} \tag{49}$$

Wherein a_0 is an elongation and a_m is the m^{th} following elongation, *both* of which are easily observed. The symbol k is the *ratio of damping.*

The observed throw, a, in equation (48) is smaller in the ratio, nearly, of $1 : \sqrt{k}$ than it would be were there no damping so that we may write $\sqrt{k} \cdot a$ for a in that equation, whence:

$$Q = \frac{i_p . T . a . \sqrt{k}}{\pi . b} \tag{50}$$

If k is very nearly unity this equation leaves only very small outstanding reduction errors.

There are three distinct kinds of error which affect the results of a set of observations. (*a*) Observational errors. (*b*) Instrumental errors. (*c*) Reduction errors. Errors of observation arise from incorrect determinations of the immediately observed quantities; Instrumental errors arise from incomplete realization of the conditions assumed in the derivation of the formulae for reduction; Reduction errors arise from the use of approximate reduction formulae, often indeed from the use of approximate formulae in the allowance for instrumental error.

Example:—Errors in the observed values of φ, ϑ and T in equation (47) lead to *observational* errors in Q. Any failure in the realization of the conditions assumed in the derivation of (47), if ignored, leads to an instrumental error in Q. The use of approximate formulae, such as (48), and (50) in the calculation of Q leads to *reduction errors* in Q.

When great accuracy is desired allowance should be made, if possible, for all instrumental error, and approximate formulae should not be used unless the reduction errors they introduce are decidedly smaller than the observational error and the outstanding instrumental error. Instrumental errors are so called from the fact that the conditions assumed in the derivation of reduction formulae refer to the arrangement and construction of the apparatus used in taking the observations. Calculations, which are made in the allowance for instrumental error, are always based upon observed values of such quantities as characterize the incompleteness in the realization of the assumed instrumental conditions.

Rigorous correction for damping.—Every actual case of vibration is damped. This damping is due to resisting forces which oppose the motion of the vibrating body. It is assumed in the following discussion that the resisting forces are strictly proportional to the velocity of the vibrating body. The errors left outstanding by this assumption can always be made inconsiderable by arranging that the vibrating body be massive; that it vibrate slowly; and, if it vibrate about an axis, that it be circularly symmetrical about that axis and smooth so as to stir the air as little as possible. With such limitations the following discussion leads to rigorous correction for damping in the use of the ballistic galvanometer.

Let ψ be the angle measured from the position of equilibrium of a vibrating body to its instantaneous position. Then, ordinarily, a torque $\mathfrak{T} = - c\,\psi - f\,\frac{d\,\psi}{d\,t}$

acts upon the body and since this torque must be equal to $I \frac{d^2 \psi}{d t^2}$, I being the moment of inertia of the body and c and f being constant, the only condition which must, in general, be satisfied by the angle ψ is expressed by the differential equation

$$\frac{d^2 \psi}{d t^2} + 2 \beta \cdot \frac{d \psi}{d t} + \gamma \psi = 0 \tag{51}$$

in which 2β is written for $\frac{f}{I}$ and γ for $\frac{c}{I}$.

Equation (51) leads, for our present purpose, to the following expression for the instantaneous value of the angle ψ :—

$$\psi = A e^{-\beta t} \operatorname{Sin} \omega t \tag{52}$$

in which A is an undetermined constant,

$$\omega = \sqrt{\gamma - \beta^2} \tag{53}$$

and t is the elapsed time reckoned from the instant of one of the passages of the body through its position of equilibrium

The time interval, T, which elapses between successive null values of ψ is called the period of the vibration and is evidently equal to $\frac{\pi}{\omega}$ or

$$\omega = \frac{\pi}{T} \tag{54}$$

Let T' be the undamped time of vibration. From (53) and (54) with the condition $\beta = o$ we have $T' = \frac{\pi}{\sqrt{\gamma}}$, whence

$$T' = T \frac{\omega}{\sqrt{\omega^2 + \beta^2}} = T \frac{1}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} \tag{55}$$

The successive elongations are maxima and minima values for ψ and the values of t at the instants of these elongations are to be found from $\frac{d \psi}{d t} = o$. Applying this method to equation (52) we easily find

$$t = \frac{1}{\omega} \operatorname{Arc} \tan \frac{\omega}{\beta} + T n , \tag{56}$$

in which n is any whole number. [At the first elonga-

tion $n = o$, at the second $n = 1$, etc.] Substituting the value of t from (56) in equation (52), we have for the first elongation ψ':—

$$\psi' = \frac{\omega}{\sqrt{\omega^2 + \beta^2}} A \, e^{-\frac{\beta}{\omega} \text{Arc tan} \frac{\omega}{\beta}} \tag{57}$$

For the second elongation ψ'',—we have similarly

$$\psi'' = e^{-\beta T} \psi'$$

also

$$\psi''' = e^{-\beta T} \psi''$$

&c. &c.

The ratio of damping, k, however, is by definition equal to $\frac{\psi'}{\psi''}$ whence

$$k = e^{\beta T}, \tag{58}$$

or

$$\lambda = \text{Log. nat. } k = \beta T. \tag{59}$$

This quantity βT, which for convenience will hereafter be represented by λ is the *logarithmic decrement* of the vibrations. Expressed in terms of the common system of logarithms it is,

$$\lambda = 2.306 \text{ Log}_{10} k.$$

Noting the relations between λ, β, π and ω; viz:—

$$\omega^2 = \frac{\pi^2 \beta^2}{\lambda^2},$$

we may evidently write equation (55) in the form

$$T' = T \frac{1}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} \tag{60}$$

The value of ψ' given in equation (57) is the observed throw of the ballistic galvanometer. If there had been no damping the throw would have been A, since ψ' reduces to A when $\beta = o$. Solving equation (57) for A and eliminating β and ω by use of (54) and (59) we have

$$A = \sqrt{1 + \frac{\lambda^2}{\pi^2}} \cdot e^{\frac{\lambda}{\pi} \text{arc tan} \frac{\pi}{\lambda}} \cdot \psi' \tag{61}$$

or

$$A = \sqrt{1 + \frac{\lambda^2}{\pi^2}} \cdot k^{\frac{1}{\pi} \text{arc tan} \frac{\pi}{\lambda}} \cdot \psi' \qquad (62)$$

Equation (61) or (62) enables us to calculate the undamped throw (A) from the observed throw (ψ').

The calculation of Q, having observed T, k, φ, ϑ, i_p is as follows :—

From the observed value of k, equation (49), λ is calculated. From the observed period T the undamped period T' [*Same as t in equation* (47)] is calculated by means equation (60). From the observed throw φ' the undamped throw A is calculated by equation (62). Then T' is substituted for t and A for ϑ in equation (47) from which Q is calculated. The steps in this calculation are tedious and cannot be greatly simplified unless approximate formulae are used.

A formula somewhat more exact than (50) is obtained as follows :—

Write $\dfrac{T}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}}$ for T in (48)

[See equation (60)].

Write $\sqrt{1 + \frac{\lambda^2}{\pi^2}} \cdot k^{\frac{1}{\pi} \text{arc tan} \frac{\pi}{\lambda}}\ a$ for a in (48)

[See equation (62)].

We thus have

$$Q = \frac{i_p . T . a . k^{\frac{1}{\pi} \text{arc tan} \frac{\pi}{\lambda}}}{\pi . b}, \qquad (63)$$

in which T is the observed period, a is the observed throw, etc. The exponent of k in (63) becomes $\frac{1}{2}$ when k is very nearly unity or λ very small; so that for this case (63) approximates to equations (50).

LECTURE IV.

Methods of Measuring the Magnetic Field.—Since, in the use of the galvanometer, the sensitiveness depends upon the value of H, that is to say, upon the strength of the earth's field, or of the artifically created field within which the needle swings, it is necessary to be able to determine the strength of this field with a high degree of accuracy.

Two of the earliest methods for the absolute determination of H were those developed by Gauss and by his co-worker in Göttingen, Wilhelm Weber. Gauss' method comprises two operations:

(1.) The determination of the rate of vibration of a suspended bar magnet.

(2.) The observation of the deflection which this magnet is capable of producing when acting upon another suspended magnet at a known distance.

From the time of vibration we get

$$T^2 = \frac{\pi^2 K}{m H}. \tag{64}$$

Where T is the time of vibration, K the moment of inertia of the magnet and m the strength of the magnet pole.

From the determination of the deflection we get

$$\frac{m}{H} = \frac{1}{2} d^3 \sin \vartheta, \tag{65}$$

or

$$\frac{m}{H} = \frac{1}{2} d^3 \tan \vartheta.$$

The latter expression applies when the deflecting magnet is stationary, in a position due east or west of the magnetometer, the former is used when the deflecting magnet is mounted on a swinging arm as in the Kew magnetometer.

Equations (64) and (65) each contain m and H. The quantity m, therefore, may be eliminated, and H may be obtained in terms which involve only the fundamental quantities—length, mass and time.

There are few operations in experimental physics which can be carried out with a higher degree of precision than this first operation of Gauss, which consists in the determination of the time of vibration. When a chronograph is accessible, it is convenient to use it in obtaining a record of the times of passage. Fig. 21 is from a portion of such a sheet, upon which have been recorded the successive transits of a needle possessing a period of five seconds. The equidistant notches are the clock records. The records of transit are marked *a*, *b*, *c*, etc.

Otherwise the period may be determined with all sufficient accuracy by the eye and ear method. In the latter case the observer counts the beats of the chronometer or clock while watching the vibrations of the magnetometer needle, and estimates to tenths of a second the successive times of transit. This estimation of tenths is a matter requiring rather more practice in the case

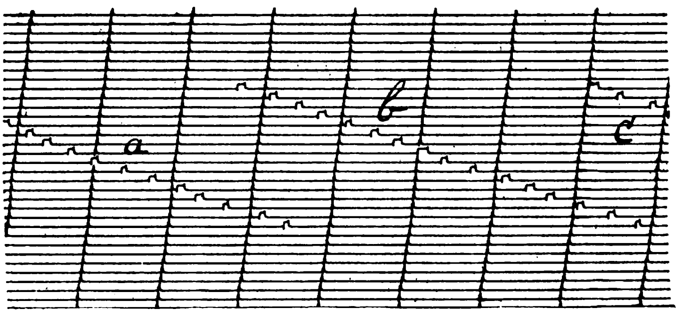

FIG. 21.

of time intervals than where one has to do with linear measurements; but it is an art readily acquired by practice. From the observed time of vibration the true period must be computed by making corrections.

(*a*) for torsion.
(*b*) for induction.
(*c*) for temperature.
(*d*) for the arc of vibration.
(*e*) for the rate of the chronometer.

Under all ordinary circumstances these corrections are individually small, but they are none of them entirely negligible.

The method of correcting for torsion is similar to that already described in the case of the galvanometer, viz.:

The ratio of the torsion of the fibre to the directive

force of the earth's magnetic field is determined by twisting the head to which the suspension fibre is attached through 90°, and noting the deflection (u) of the magnet.

This ratio* is

$$\frac{H}{F} = \frac{u}{90^\circ - u}. \tag{66}$$

The coefficient of induction (μ), by means of which the earth's inductive action in changing the magnetic condition of the needle or bar is expressed, is the increase in magnetic moment due to the action of a unit inducing force.

Lamont's method of determining μ consists in determining the deflection produced by the magnet to be tested is placed vertically, first, with the north seeking pole upwards, and then downwards at a constant distance from a suspended needle. Under these conditions equation (65) becomes respectively

$$\frac{m - V\mu}{H} = \tfrac{1}{2}\, d^3 \sin \vartheta, \tag{67}$$

and

$$\frac{m + V\mu}{H} = \tfrac{1}{2}\, d^3 \sin \vartheta'; \tag{68}$$

where V is the vertical component of the directive force of the earth.

By the combination of (67) and (68) we obtain

$$\frac{V\mu}{m} = \frac{\sin \vartheta' - \sin \vartheta}{\sin \vartheta' + \sin \vartheta} = \frac{\tan \frac{1}{2}(\vartheta' - \vartheta)}{\tan \frac{1}{2}(\vartheta' + \vartheta)}. \tag{69}$$

By making use of the relation $V = H \tan i$, in which i is the magnetic inclination, and of the equation $\frac{m}{H} = \frac{1}{2} \sin u$ which expresses the action of the magnet when used as a deflecting bar at unit distance, we may write

$$\mu = \frac{\sin u}{2 \tan i} \cdot \frac{\tan \frac{1}{2}(\vartheta' - \vartheta)}{\tan \frac{1}{2}(\vartheta' + \vartheta)}. \tag{70}$$

The induction coefficient is a small quantity. It is given by Figg and Whipple, in the case of Kew magnetometer No. 47, for example as 0.0000050. It is usually determined once for all by the method just described.†

* In this case and in the discussion of the subsequent corrections the notation is in the main that given by the Kew Observatory in the official sheets.

† See Stewart and Gee: Practical Physics, II., p. 488.

The temperature correction is more important. The bar to be tested is placed in a bath, its axis east and west. Its deflecting power upon a magnetometer needle suspended at a suitable distance is noted at various temperatures throughout a range of about 40° *C.* The influence of a rise of temperature upon the strength of magnetization is always to diminish it, and the change can be expressed by an equation of the form :

$$m_t = m_0 (1 - a\,t - b\,t^2). \tag{71}$$

where m_t is the magnetic moment at the temperature

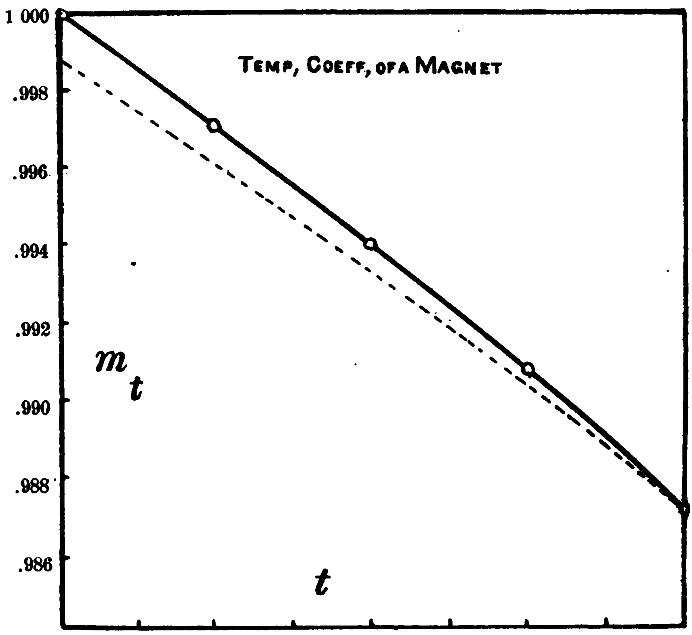

FIG 22.

of observation, m_0 that at 0° *C*, and a and b are coefficients. The values of a and b vary somewhat with individual magnets. The following are average coefficients:

$$a = 0.000289.$$
$$b = 0.00500077.$$

This effect of temperature is not wholly temporary. If, for example, measurements be made upon a magnet, giving for rising temperatures the curve *I*, Fig. 22, the succeeding curve for falling temperatures (*II*) will

not coincide with it, but will lead to a lower final value m_0'. The amount of this divergence depends upon the age, temper and previous history of the magnet.

The corrections for the arc of vibration are made by means of the well-known formula

$$T = T_{obs}\left(1 - \frac{s}{86400} - \frac{a\,a'}{16}\right). \qquad (72)$$

In this expression s is the number of seconds gained in a day by the chronometer or clock, and a, a' are the initial and final values of the semi-arc of vibration.

The second operation of Gauss may be carried out in two ways: the first of which is called the method of sines; and the second, the method of tangents. The method of tangents is the more convenient in cases in which the determinations are made repeatedly in a

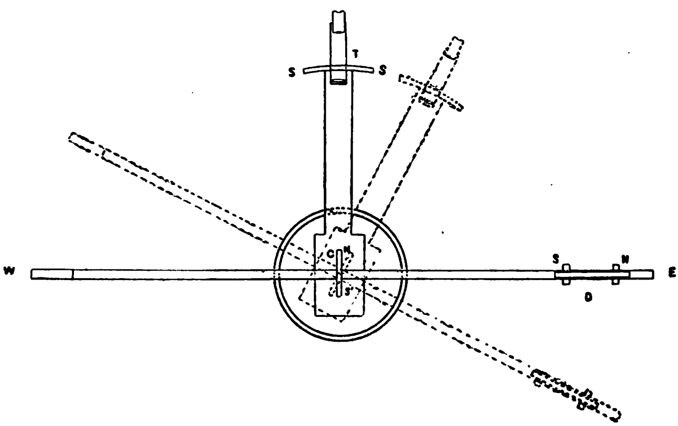

FIG. 23.

single locality; the method of sines, on the other hand, is better adapted for portable instruments. Of such instruments, the Kew magnetometer is the best known. Its essential features are a central box or house, containing the suspended magnet C (Fig. 23), an arm extending to the north of the axis of the instrument upon which is mounted a small reading telescope (t) bearing a short circular scale (s). By means of this, observations are made upon the position of the magnetometer needle. A second bar accurately graduated, as to length, extends to the east and west at right angles to the arm carrying the telescope. Upon this, at fixed points, is placed, for the purpose of the second operation, the deflecting magnet (D). The entire instrument, includ-

ing the two arms just mentioned and the magnetometer box itself, are capable of rotation upon the vertical axis, which axis corresponds with that of the suspended magnet. The angles, through which this instrument is turned, are measured upon a horizontal circular scale similar to that with which sine galvanometers are provided; and the method of making the readings is the same.

The expression made use of in the second operation of Gauss, see equation 65, is derived as follows:

Consider a magnet, *D* (Fig. 23), of strength of pole,

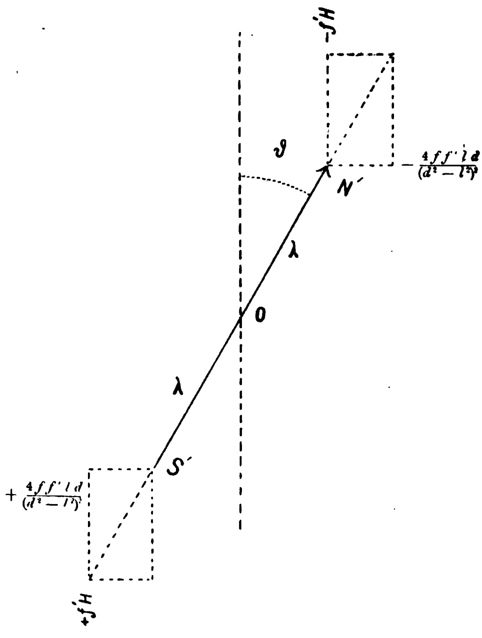

FIG. 24.

f, acting to deflect the suspended magnet, *C*, the strength of pole of which is f'.

The action between pole *S* and pole N' is

$$\frac{-ff'}{(d-l)^2},$$

in which *d* is the distance between the centres of *D* and *C*, while *l* is half the distance between the poles of the former.

The action between N and N' is, however,

$$\frac{+ f f'}{(d + l^2)},$$

and the total force upon N' (Fig. 24), is

$$f f' \left(\frac{1}{d + l^2} - \frac{1}{(d - l)^2} \right) = \frac{-4 f f' l d}{(d^2 - l^2)^2}. \tag{73}$$

The total force upon S' is, in the same way,

$$\frac{+ 4 f f' l d}{(d^2 - l^2)^2},$$

and the couple due to the deflecting magnet is

$$\frac{8 f f' l \lambda d \cos \vartheta}{(d^2 - l^2)^2},$$

when D remains fixed in the east and west direction, or

$$\frac{8 f f' l \lambda d}{(d^2 - l^2)^2}$$

for the case of the Kew magnetometer.

When the suspended magnet has come to rest at the deflection ϑ, we have

$$\frac{8 f f' l \lambda d}{(d^2 - l^2)^2} 2 = f' \lambda H \sin \vartheta. \tag{74}$$

or

$$\frac{2 f l}{H} = \frac{(d^2 - l^2)^2}{2 d} \sin \vartheta. \tag{75}$$

Now the magnetic moment of the deflecting magnet is

$$m = 2 f l,$$

and when l is small compared to d (a condition which should always be fulfilled in performing this operation) we may use the approximation $(d^2 - l^2 = d^4$, and reduce (75) to the same form as (65), viz.:

$$\frac{m}{H} = \frac{d^3}{2} \sin \vartheta.$$

The following are the corrections to be applied in the computation of the results of the second operation :

(*a*) for temperature of the bar holding the deflecting magnet.

(*b*) for the distribution of magnetism in the same.

(*c*) for the temperature of the magnet.

(*d*) for induction.

For the first of these it is sufficient to assume an average coefficient for brass and to correct the observed value (d') to the proper value (d_0) by noting the temperature at which the observation is made, and the interval to the temperature at which the scale upon the bar is right.

The form is

$$d' = d_0 (1 + 0.000018 (t' - t_0). \tag{76}$$

The correction for distribution is obtained by making two sets of readings of deflection with the magnet D at very different distances from C. The most favorable relation to be that in which $\frac{d_1}{d_2} = \frac{3}{4}$.

The correction for distribution (ρ) is applied by means of the formula

$$\frac{m}{H} = \left(\frac{m}{H}\right)' \left(1 - \frac{\rho}{d^2}\right). \tag{77}$$

Where $\frac{m}{H}$ is the corrected and $\left(\frac{m}{H}\right)'$ the observed value.

ρ is determined thus:

$$A_1 = \tfrac{1}{2}\, d_1^3 \sin \vartheta_1$$
$$A_2 = \tfrac{1}{2}\, d_2^3 \sin \vartheta_2. \tag{78}$$

in which A_1 and A_2 are the two observed values of $\frac{m}{H}$ at distances d_1 and d_2.

From (77) it follows, however, that

$$A_1 \left(1 - \frac{\rho}{d_1^2}\right) = A_2 \left(1 - \frac{\rho}{d_2^2}\right). \tag{79}$$

or

$$\rho \left(\frac{A_1}{d_1} - \frac{A_2}{d_2}\right) = A_1 - A_2. \tag{80}$$

From equation 80, which contains only d_1, d_2, ϑ, and ρ, the correction value can be computed numerically. The expression (77) is only an approximation, it is true, but since the value of ρ is very small, it is not necessary to use the higher terms which appear in a more accurate formula.

The methods of obtaining the temperature coefficient (q) and the correction for induction (μ), have already been indicated. It should be noted, however, that μ depends upon the position of the magnet D in the field,

as well as upon H. In the tangent method of Gauss it disappears altogether. In the sine method, where the axis of D makes a final angle ϑ with the east and west direction, the inductive effect on D is $\mu H \sin \vartheta$.

From this expression we may obtain an approximate form by use of the equation $\sin \vartheta = \frac{2m}{H d^3}$.

The form commonly used is

$$m = m_0 \left(1 + \frac{2\mu}{d^3}\right) \tag{81}$$

in which m is the corrected moment and m_0 the observed value.

Kohlrausch's Method for H.—Next in importance to Gauss' method comes that of Kohlrausch, where absolute determinations of H are desired. Kohlrausch's method, indeed, in those cases in which the knowledge of H is to be used as a factor in the constant of the galvanometer, is much to be preferred to any other, because it permits of the determination of H in the precise region to which the value is to be applied; whereas in the case of the Kew magnetometer it is oftentimes necessary to make determinations which are strictly applicable only to regions distant several feet from the precise locality from which we desire to know the intensity of the field. The Kohlrausch apparatus consists of the tangent galvanometer, which may be, and should be, the galvanometer for the calibration of which the value of H is desired, and a swinging coil. The coil is held vertically in the magnetic meridian by means of a bi-filar suspension. The suspension consists of two wires which serve to carry current into and out of the suspended coil, and at the same time give it the necessary directive force. This coil should be suspended as nearly as possible in the region containing the needle of the galvanometer which we wish to calibrate. In the case of tangent galvanometers of the Helmholtz pattern, it is oftentimes entirely practicable to mount the swinging coil midway between the fixed coils of the galvanometer, so that the needle will be within the plane of the former and in its axis. Such an arrangement exists in the large tangent galvanometer described in the first lecture. The method of procedure is as follows: A current i is sent through the coils of a galvanometer of known dimensions, and in the case in question this should be the galvanometer to be calibrated. We have then

$$i = \frac{H}{G} \tan \vartheta. \tag{82}$$

The same current is then sent through the suspended coil. If the galvanometer coils or the suspended coils are of appreciable resistance, as compared with the remainder of the circuit, it is necessary to have elsewhere resistances which can be inserted and removed, the values of which have been previously adjusted so as to correspond precisely to the resistance of the galvanometer and of the swinging coil respectively. A convenient arrangement for doing this is represented in Fig. 25, in which R_g and R_s are resistances, non-inductively wound, which are to be substituted in turn for the galvanometer and for the swinging coil by means of the switch S. When the current i is sent through the swinging coil it will deflect the latter. This deflection is to be read by means of a suitably adjusted telescope and scale. We have then

$$i = \frac{M_b}{A\,H} \tan \alpha, \tag{83}$$

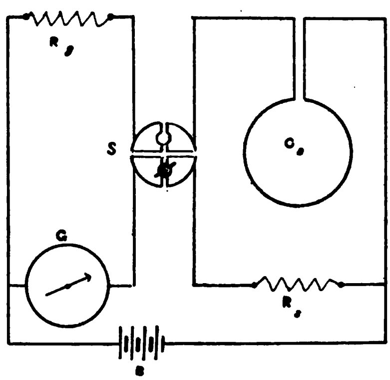

FIG. 25.

in which M_b is the moment of the bi-filar suspension, A is the effective area of the suspended coil, the dimensions of which must have been determined by previous measurement, and α is the deflection of the coil from the magnetic meridian. By combining equations 82 and 83, we obtain the following expression for H^2 in terms of the constants of the galvanometer and of the swinging coil, the moment of the latter and the ratio of the tangents of the deflections, viz.:

$$H^2 = \frac{G\,M_b}{A} \frac{\tan \alpha}{\tan \vartheta}. \tag{84}$$

In the hands of Kohlrausch this method has afforded some of the most precise determinations of the abso-

lute value of the horizontal component of the earth's magnetic field which have ever been made. He availed himself of it, for example, in connection with his re-determinations of the electro-chemical equivalents of silver and copper. Such operations demand the most precise knowledge of all the factors which enter into the operation.

The most troublesome features of this method are those which deal with the temperature changes of M_b, the secular changes of the same and variation in the resistance of the suspension wires. These wires to give sufficient delicacy to the method must be of small size, and they must consequently be subjected to high current densities.

In the case of the large tangent galvanometer at Cornell University, reference to which was made in the first lecture, an arrangement was perfected for the determination of H by a modification of the method of Kohlrausch. A large swinging coil, the diameter of which was one meter, was suspended by means of a phosphor bronze wire two meters long, the upper end of which was attached to a torsion head carrying a circle and vernier. The method consists in bringing the swinging coil back to its zero position by twisting the suspension wire. The position of the coil was read by means of a telescope and scale at a distance of three meters to the south of the instrument, for which purpose the torsion head could be given a slow motion of rotation by means of a tangent screw operated by the observer at the reading telescope. The suspension wire served also to introduce current to the suspended coil, which consisted of 100 turns of No. 18 copper wire. The other terminal of the coil consisted of a wire situated in the axis of rotation, the end of which was dipped in a mercury cup at the base of the instrument. This form of suspension gave greater delicacy than could be obtained by means of any bi-filar suspension, which would be capable of carrying the currents which it was necessary to introduce into the coil. The method possessed also the advantages common to what are known as *zero methods*. The equations, by means of which H is determined with this instrument, differ from those which apply to the Kohlrausch method only in two particulars. In the first place, the force of torsion used in returning the wire to its original position is proportional to the angle through which the suspension wire is twisted. In the second place, we have to substitute for the moment of the bi-filar suspension the moment of torsion of the wire. Making these changes in equation 83, we have the following :

$$i = \frac{M_t}{A\ H} a, \tag{85}$$

$$H^2 = \frac{G\ M_t}{A} \cdot \frac{a}{\tan \vartheta}. \tag{86}$$

The moment of torsion is determined by substituting for the swinging coil, which is so adjusted as to be readily unmounted and removed from its position, a cylindrical brass weight the moment of inertia of which can be determined directly from its mass and from its dimensions. This cylinder, in the instrument under consideration, weighs 4954.22 grammes, and its diameter is 7.8747 cm. The weight is hung in place of the coil and its period of oscillation is accurately determined by the aid of the chronograph. The expression for the moment of torsion takes the usual form of the equation for the torsion pendulum, viz.:

$$M_t = \frac{4\ \pi^2\ K}{T^2}. \tag{87}$$

in which, when we know, the moment of inertia K and the period of oscillation T, we have all the factors necessary to the computation of H in absolute measure. Substituting in equation (86) the above value of the moment of torsion, we have

$$H^2 = \frac{4\ \pi^2\ K}{T_2} \cdot \frac{G}{A} \frac{a}{\tan \vartheta} \tag{87}$$

or

$$H^2 = C \frac{a}{T^2 \tan \vartheta}. \tag{88}$$

The quantity C in equation (88) is a constant such that :

$$C = \frac{4\ \pi^2 K\ G}{A}. \tag{89}$$

When this method was first put into operation in 1886, two serious sources of error, one of which was entirely unexpected, arose. The first of these was an error due to the influence of temperature upon the moment of torsion of the suspension wire. This error has its basis in a property of matter which is perfectly well known. It is not a difficult matter to determine once for all the temperature coefficient of torsion for the material used, and to apply the correction. The difficulty in maintaining a vertical wire, two meters long, at anything approximating a constant temperature throughout its entire length, however, was found to be unsurmountable under the conditions which existed in the observatory where the galvanometer was situated,

and no satisfactory correction for the temperature of the wire was reached until after many expedients had been tried and abandoned; the following method of ascertaining the average temperature of the wire at the precise time when each observation was made came to be adopted.

This method of integrating the temperature for the entire length of the wire consisted in placing a No. 40 copper wire parallel to the suspension and as close to the same as could be without actual contact. This copper wire was drawn back and forth several times. It was placed in series with a compensated resistance, the value of which was approximately the

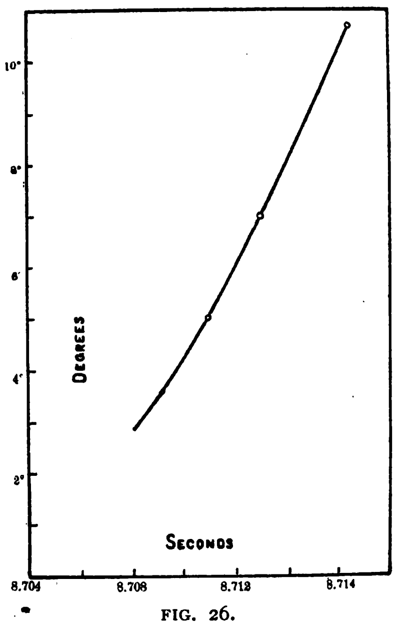

FIG. 26.

same as its own and a suitable current sent through the circuit containing the two. A sensitive galvanometer was mounted in another part of the observatory, by means of which the flow of potential through the compensated resistance and through the temperature wire just described could be compared. The ratio of these deflections gave the average temperature of the wire with a high degree of accuracy.

Before mounting the fine copper for this purpose, its temperature coefficient had been determined, and the ratio of the deflections when the galvanometer was

shunted across its terminal, to that obtained when the galvanometer was shunted across the terminals of the compensated resistance, had been ascertained for a sufficient range of temperatures. By the use of this simple device, the difficulties arising from difference of temperatures in the suspension wire were eliminated.

The other source of error was of a more serious character. It was found that the torsional elasticity of the suspension wire varied continually with age. The change, which was very marked, indeed, at first, diminished slowly as time passed; but it never became a negligible quantity. The time curve of this wire has been taken with great care, and it now covers an interval of nearly ten years. By means of this curve the moment of torsion of the wire for any desired date can be ascertained with a sufficient degree of accuracy; but with-

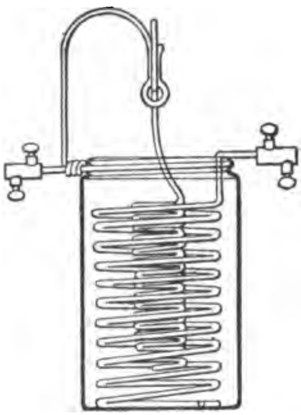

FIG. 27.

out this correction the values for H determined by this method would be seriously at fault. The character of the first of these two variations, that due to temperature and is shown graphically in Fig. 26, which gives the relation of the rate of vibration of the wire when attached to the calibration weight already described. This curve was made on February 26, 1887, and is from measurements by Professor H. J. Ryan. Determinations at later dates would afford data showing a slower period. The rate at 10°, on October 28, 1889; for example, according to measurements by Mr. N. H. Genung was 8.689 + seconds.

To control these factors, upon which accuracy depends, is a matter of considerable difficulty, and the method of Kohlrausch for the determination of H is rendered a laborious one because of them.

The Determination of H by Means of the Copper Voltameter.—For all ordinary operations with the tangent galvanometer, a sufficiently accurate determination of H can be obtained by a method which is much more convenient than those of Gauss or Kohlrausch. This method consists in sending through the galvanometer a current, the intensity of which is measured by means of copper voltameters placed in the circuit, and noting the deflection produced. The requisites are a steady source of current, such as a storage battery of considerable capacity, a fine balance, a fairly accurate timepiece and a copper voltameter of proper construction. The form of voltameter which has shown itself best adapted to accurate work is one which is at the same time the most easily constructed. I refer to the spiral coil voltameter of Professor Ryan.* This consists of two suitable jars containing a slightly acidulated solution of the sulphate of copper, two coils of pure copper wire about five centimeters in diameter which are to form the losing electrodes, two coils of smaller diameter

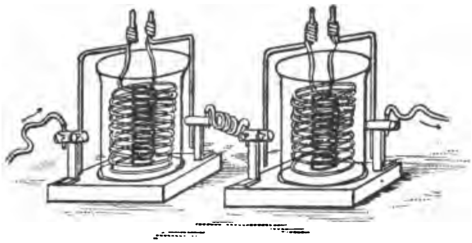

FIG. 28.

constructed from the same wire, which are to constitute the gaining electrodes, and any simple device for holding these coils pair-wise in the cells with a common vertical axis corresponding to the axis of the jar (see Fig. 27). To construct these coils, it is only necessary to take a few feet of copper, size No. 10 or No. 12, to strip the same of its isolation, to clean the wire thoroughly by clamping one end of it and drawing sand paper, grasped in the hand, briskly, over its entire length several times. The wire is then wound upon cylinders of suitable diameter so as to form two large and small coils such as have been already described. When completed, these coils will have a length along the axis of about three inches. and their diameters will be respectively for the losing coils, five, and for the gaining coils, two centimeters.

The smaller coils are carefully weighed upon a balance of high precision, are then mounted within the

large coils in the two jars, Fig. 28, and concentric with the same; the jars are filled with the electrolytic solution, electric connections are completed, and the time of making circuit is carefully noted. In the course of a half hour or thereabouts, during which the current is flowing through the cells and through the galvanometer, a number of readings of the deflection are taken. The current is then broken at a time accurately noted, and the inner or gaining coils are removed from the voltameter. They are rinsed with water and then with alcohol, after which they are dried without friction with filter paper, or by holding them at a safe distance over the flame of a bunsen burner.

If the operation has been a successful one, the surface will possess a uniform and beautifully tinted surface characteristic of freshly deposited electrolytic copper. Any marked granulation of the surface would indicate too great a current density, and would subject the results of the measurement to suspicion. Under such circumstances the calibration should be repeated. The amount of copper deposited upon two plates, as shown by the comparison of the weighings before and after should agree to within two-tenths of one per cent. Properly carried out, therefore, this method will give the value of H to a like degree of precision.

LECTURE V.

Galvanometers with Artificial Fields.

1. *Instruments with Strong Fields*.—The magnetic field of the galvanometer is frequently strengthened artificially for one or more of the following reasons:

(*a*) To increase the constant $\frac{H}{G}$, thereby securing an instrument suitable to the measurement of heavy currents.

(*b*) To diminish the period of vibration of the needle.

(*c*) To obtain immunity from magnetic disturbances, such as the daily fluctuations which take place in the value of H, and the accidental variations brought about by the proximity of masses of iron or by the inductive influence of the dynamo motors, and of line wires carrying current.

The most important instruments of the kind under consideration are the galvanometers of the D'Arsonval type. In these well-known galvanometers a strong field is obtained by means of a nearly closed magnetic circuit. Within the air space of this magnetic circuit is placed a coil of wire, through which the current to be measured is allowed to pass. This coil has freedom of rotation upon an axis at right angles to the lines of force, and also at right angles to the axis the coil itself. In order to hold such a coil in place in the very strong fields which are made use of in these instruments the suspension is by means of wires vertically fastened above and below.

The original type described by Deprez and D'Arsonval,* is shown in Fig. 29. A coil thus held between tense suspension wires vibrates rapidly, and a short period of oscillation is, therefore, one of the characteristic features of such instruments.

The D'Arsonval galvanometer has been subjected to a great variety of modifications. We have, for example, the moving coil without an iron core, a moving coil

* Deprez and D'Arsonval: Comptes Rendus 94, p. 1347, 1882.

with an iron core, a moving coil the interior of which is filled with a stationary piece of soft iron. The field in which the coil swings is sometimes that of a permanent magnet of the horseshoe type, sometimes that of an electromagnet, and sometimes that of a solenoid without iron. The air gap also is of various sizes, from the very large air gap of the Thomson graded galvanometer to the exceedingly small one employed in instruments of the Breguet form. The advantages

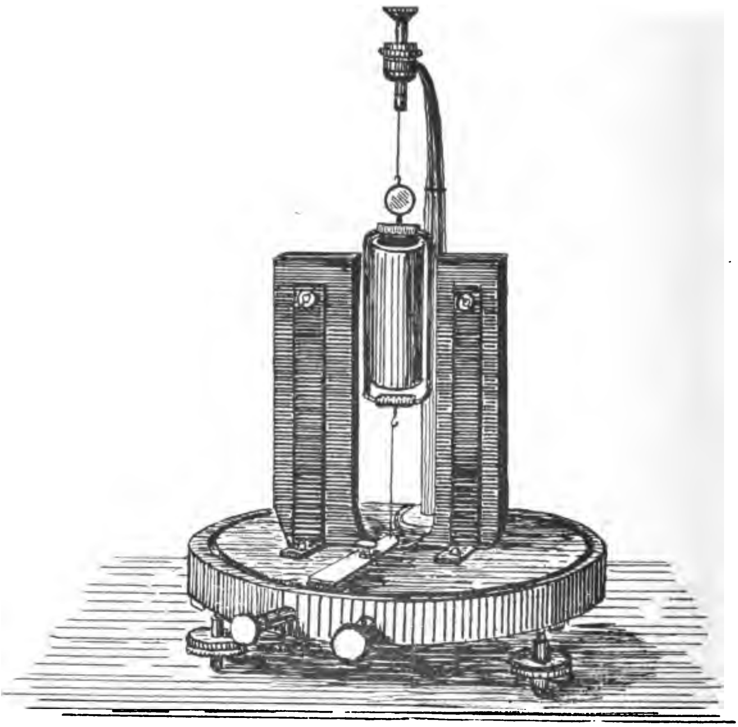

FIG. 29.

of all these galvanometers may be summed up in the statement that they are exceedingly quick of action and remarkably free from outside influences. As to the permanency of their indications, it is evident that any instrument, the constant of which depends upon the maintenance of an unchanged field due to permanent magnets, must be subject to a certain amount of secular change. Whether the time change in the field of such instruments can be reduced to an inappreciable quan-

tity is a subject about which there has been considerable discussion. Dr. Koepsel,* for example, in a paper read before the Electro-technical Congress at Frankfort, in 1891, took the ground that the use of steel magnets in instruments for the measurement of electric current should be altogether abandoned on account of the lack of permanence. This view has been combatted, however, on the part of those who have had much experience in making instruments in which permanent magnets are used. The permanence of such magnets, undoubtedly, depends in part on the size of the air gap, and increases as the latter is reduced.

Instruments such as the Thomson graded galvanometers, on the one hand, in which the magnetic circuit is nearly half through the air, exhibit much more rapid decadence than instruments of the Deprez type in which the air gap is reduced to a minimum. A graded galvanometer with a home-made magnet which had been constructed to take the place of the original magnet belonging to the instrument showed, for example, a change of constant in one year from 7.00 to 6.61. This marked falling off in the strength of the field may, with justice, be ascribed in part to the inadequate treatment of the permanent magnet in preparing it for use in such an instrument. The original magnet belonging to a similar galvanometer showed, however, a scarcely better record. The constant fell off in this second case from 4.52 to 4.44 in one year. An ammeter of the D'Arsonval-Deprez type showed somewhat greater permanence. A current, which, on the 30th of November, 1892, produced a deflection of

36.2 scale divisions to the right,
35.8 scale divisions to the left,

was found in October, 1893, to produce, respectively

35.1 scale divisions to the right,
34.9 scale divisions to the left.

As originally constructed, the calibration curves of the D'Arsonval galvanometer were by no means straight. Figure 30 shows the curve for right and left deflections in the case of the ammeter just referred to. It is obvious, however, that the law of deflections in all such instruments is under control by modifying the shape and disposition of the pole pieces. Professors Ayrton and Perry have shown that by this device the curve can be readily straightened.

An interesting example of the application of the principles upon which galvanometers with strong fields de-

* Koepsel: Verhandlungen des internationalen Elektrotechniker-Congresses zu Frankfurt, 1892, Zweite Halfte, p. 3.

pend is found in the Moler curve writing voltmeter.* This is essentially a galvanometer of the D'Arsonval type in which a needle of soft iron is mounted in the strong field between the poles of a powerful permanent magnet.

The needle carries a short aluminium pointer which records its oscillations upon the smoked drum, Fig. 31. The vibrations of the needle of this instrument are so rapid that by means of it one can follow the fluctuations of current during a single revolution of a dynamo or motor. The amplitude of vibration is necessarily quite small, but the indications of the instrument are nevertheless exact, and when measured and duly magnified they are found to correspond excellently with results obtained by other methods.

It is possible, by means of an instrument of this kind, to make interesting studies of a great variety of pheno-

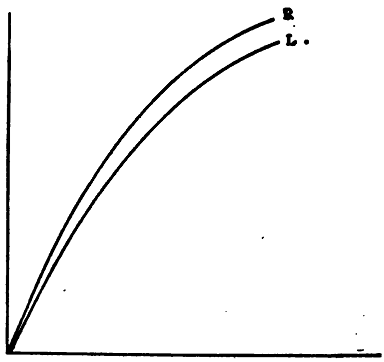

FIG. 30.

mena in which rapid fluctuations of current occur. The instrument was originally devised for the purpose of exploring the field of dynamos and motors, a purpose to which it is well adapted. It can, however, be used in many other ways.

Fig. 32 shows the tracing obtained by means of this instrument in the study of the performance of arc lamps. The voltmeter was placed across the terminals of a direct current lamp the construction of which is such that the carbons are held apart by a spring until they are brought together by the action of the current through the shunt coil, after which they are separated again and the arc is formed. The point marked *w* in the figure is that at which the circuit was closed. The vertical distance between the upper and lower tracings at

* American Institute of Electrical Engineers, vol. 9, p. 223.

b is 50 volts which is the normal potential difference of the lamp. It will be seen that immediately after closing the circuit at w there is a rise of potential to a much higher value during the interval before the shunt coil comes into operation; furthermore, that the carbons do not come into contact for a considerable time, viz., that which elapses between the point marked w and that

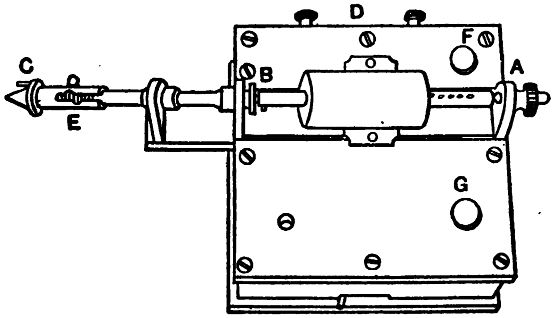

FIG. 31.

marked x. There is also a further interval of time from x to y, during which the contact between the carbons, which had been poor at first, improved as the tips grew hot. This appears from the fact that the potential difference continues to fall off until y is reached, at which time it is virtually reduced to zero. Finally, the adjusting mechanism of the lamp begins to act, and the carbons are drawn apart to their normal condition.

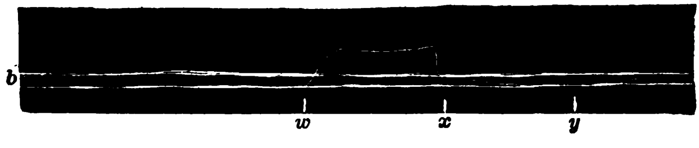

FIG. 32.

2. *Instruments with Weak Fields.*—A more important modification of the field of the galvanometer than that which we have just considered, is in the direction of reducing its intensity. The object sought in such cases is increase of sensitiveness. The limit to which such increase can be carried is reached only when the natural or artificial changes of the field thus produced become

so great as to cause troublesome drifting of the galvanometer needle. A magnetized needle in the weakened field shows motions corresponding to those of the declination needle, but the amplitude of fluctuation is increased. The sensitized galvanometer, therefore, drifts more and more as its sensitiveness increases.

The following simple demonstration of the relation between sensitiveness and the fluctuation of the direction of the resultant force of the weakened field is due to Mr. F. J. Rogers.* In Fig. 33, let O E represent the direction and the directive force of the earth's magnetic field which has been partly counteracted by the introduction into the neighborhood of a controlling magnet which produces a field represented by the line O M. The resultant field is O R. If now from any cause the

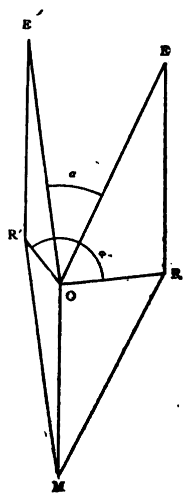

FIG. 33.

original field be subjected to a change of direction and intensity that it must be represented by O E′, which makes an angle a with O E, the new resultant formed by the combination of O M with O E′ will take a new direction O R′, which makes a much larger angle a' with the first resultant O R. To the same writer is due a very interesting experimental study of the behavior of sensitive galvanometers.

For this purpose two galvanometers were taken, one of which was highly sensitized by the action of a controlling magnet, while the other possessed a field due to the uncounteracted intensity of the earth's magnetism at the point at which the instrument was set up. The

* F. J. Rogers, *The Crank*, vol. 6, 1892, p. 270.

first of these was studied on three successive days for the purpose of observing the range of fluctuation of the zero point under the ordinary changes in the direction of the earth's lines.

The course followed by the needle is shown in Fig. 34. It was the same in all essential particulars during the three days in question, reaching a maximum of elongation from its mean position daily just before 2 P. M. These curves correspond very closely with those which would be obtained by the observation of the movement of a declination needle at times when there was no marked magnetic storm. The amplitudes of oscillation, however, are very much greater owing to the weakness of the field. On each of these days the other galvanometer, the needle of which was suspended in the earth's field, went through the same range with about one-twentieth of the amplitude shown in Fig. 34.

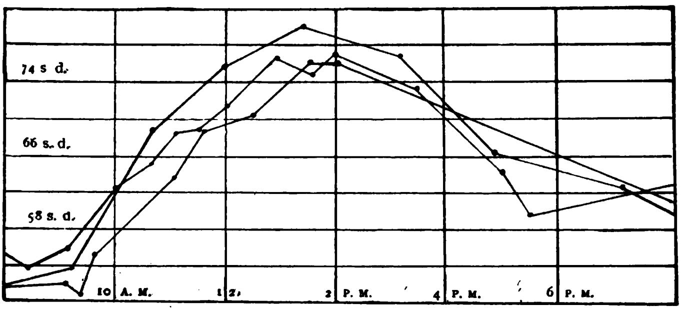

FIG. 34.

To further establish the relationship between the movements of a sensitive galvanometer and those of the declination needle, in other words, to ascertain whether the movements, familiar to all who use sensitive instruments of the class in question, are due to local disturbances, or to those widespread magnetic fluctuations which cause magnetometer needles over the entire continent to move together, observations were made with the two instruments already described upon a day when there was marked magnetic disturbance. The result is shown in Fig. 35.

In this diagram the amplitude of fluctuation of the needle, which was suspended in the earth's field, was multiplied by a constant factor such as to bring it to the same scale as that of the more sensitive galvanometer. To make sure that the fluctuations which affected the two galvanometers simultaneously through

the day, although they were mounted in different rooms of the laboratory, were not due to local causes, but were the result of changes in the magnetic field of the earth itself, records were obtained for the day in question from the magnetic observatory at Washington. These were multiplied by the proper factor to bring them to the exaggerated scale applicable to the sensitized galvanometer, and were plotted upon the same sheet. In Fig. 35, to which reference has just been made, the unbroken line represents the changes in the position of the declination needle as recorded at Washington on the day in question. The two broken curves are those obtained by making readings with the two galvanometers in the laboratory at Ithaca. It will be seen that the three curves agree in a remarkable manner throughout.

The consideration of these results makes it obvious that in order to use a sensitive galvanometer in operations of precision, two things are necessary:

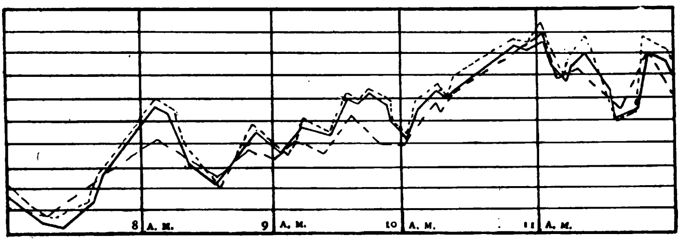

FIG. 35.

1. A knowledge of H in the locality where the instrument is in use at the time when calibration of the latter is made.

2. Some method of following the fluctuations of H from moment to moment.

A discussion of the means of meeting this second requirement will be given in a subsequent lecture.

The methods of determining H, described in the fourth lecture are very laborious, and it is desirable, therefore, to substitute for them some means for comparing the strength of unknown fields with that of known fields previously determined for this purpose the method of Wilhelm Weber is most convenient. It is described below.

Weber's Method for the Determination of H.—The procedure consists in turning a coil of known dimensions (the earth inductor) suddenly through 180°, and noting

the throw of the slow moving needle of a ballistic galganometer placed in circuit with this coil. Fig. 36 shows the instrument in its simplest form, while Fig. 37 gives the arrangement of the electrical circuit.

The earth inductor is a coil of considerable area, and consisting of many turns of copper wire. The dimensions of the coil, the number of turns and the resistance of the instrument will depend upon the character of the galvanometer with which it is to be used.

The coil is mounted in a strong wooden frame with

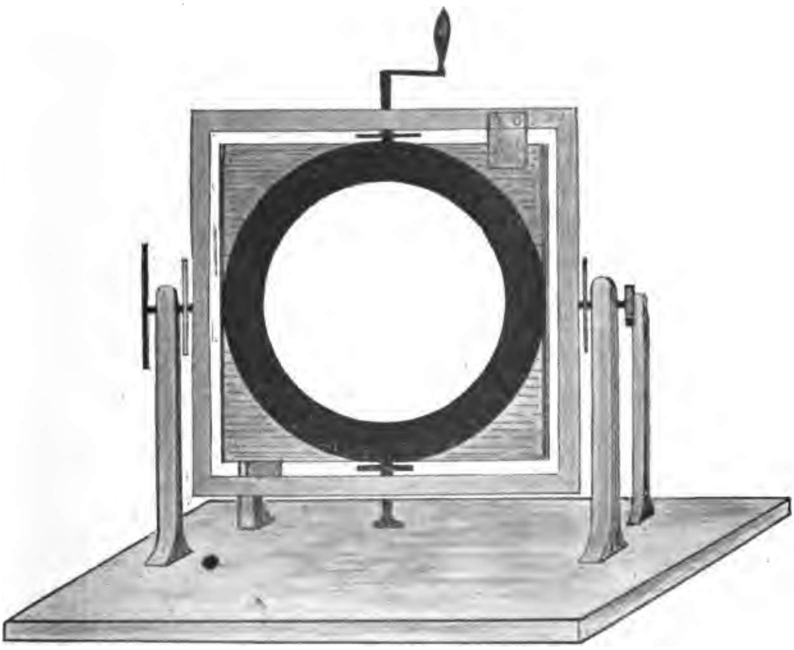

FIG. 36.

freedom to revolve upon a vertical axis. Stops are so placed as to limit this motion to exactly 180°. The frame itself is free to turn upon a horizontal axis, with stops to facilitate the adjustment of it in a horizontal or in a vertical plane.

The base of the earth inductor should be provided with a good level, and with levelling screws. In the operations to be considered here it is desired to compare the value of H in a locality where that quantity is known, as, for example, in the magnetic observatory,

with the value of H where the galvanometer is mounted. For this purpose the earth inductor is carefully levelled in the former locality with its coil vertical and against the stops, the axis of the coil being in the magnetic meridian. It is connected with a line leading to the galvanometer. The earth inductor is in series with the latter and with a resistance R. These three portions of the circuit, viz., the coils of the galvanometer of the earth inductor and of the resistance R, should include nearly all the resistance, that of the line being negligible.

The circuit having been completed, the observer watches the galvanometer while an assistant swings the earth inductor through 180°. A series of readings are thus made, using the galvanometer ballistically.

It is necessary to accuracy, that the period of vibration of the instrument be much longer than the interval of time occupied by the semi-revolution of the inductor. Otherwise one of the conditions indicated in Lecture III, viz., that the entire quantity of electricity due to the motion of the coil should traverse the coils of the galvanometer before the needle had moved through a considerable angle, will not be fulfilled.

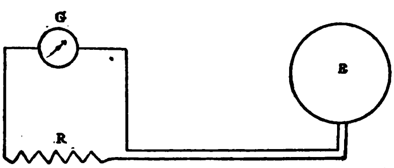

FIG. 37.

After the completion of a series of readings the earth inductor is removed to the spot in which the intensity of the field is to be compared with that of the locality which that instrument had occupied. After proper adjustment of the coil in its new place, a new series of semi-revolutions is made and the corresponding deflections are noted.

It is by the comparison of these deflections that the two magnetic fields are brought into relative measurement. The changes of the field in the second locality can, moreover, be followed by repetition from time to time of the second series of determinations.

The expression for quantity of electricity generated by the motion of the earth inductor is

$$Q = \frac{G\, H_g\, T}{\pi} 2 \sin \tfrac{1}{2} \vartheta = 2\, G\, H_g \sqrt{\frac{K}{H_g\, M}} \sin \tfrac{1}{2} \vartheta \quad (90)$$

an equation, the development of which has been given

in the lecture on the ballistic galvanometer. The quantity Q, however, depends upon the area A of the earth inductor coil, and the resistance R of the entire circuit as well as upon the strength of the field within which the rotation of the coil occurs. This relationship is expressed by the formula

$$Q = \frac{2\,H_e\,A}{R}, \tag{91}$$

combining equations 90 and 91 we may write

$$\frac{H_e\,A}{R} = \frac{G\,H_g\,T}{\pi} \sin \tfrac{1}{2}\,\vartheta. \tag{92}$$

This equation may be used in several ways:

1. Given H_e, A, R, G and T,

H_g, the strength of the field in which the galvanometer needle swings may be determined.

2. Given the constant of the galvanometer in the field H_g also A and R, any field H_e in which the earth inductor is turned may be computed in absolute measure.

3. The operations just described yield two equations of the type 90, viz.:

$$\frac{H_e\,A}{R} = \frac{G\,H_g\,T}{\pi} \sin \tfrac{1}{2}\,\vartheta_e. \tag{93}$$

$$\frac{H_x\,A}{R} = \frac{G\,H_g\,T}{\pi} \sin \tfrac{1}{2}\,\vartheta_x. \tag{94}$$

By combining these we are able to eliminate all the constants, and to obtain a ratio between the fields to be explored, viz.:

$$\frac{H_e}{H_x} = \frac{\sin \tfrac{1}{2}\,\vartheta_e}{\sin \tfrac{1}{2}\,\vartheta_x}. \tag{95}$$

The method of the earth inductor is especially useful when a very strong field, such as that which exists in the air gap of an electromagnet, is to be compared with H_e.

For such measurements a coil should be constructed, the cross-section of which is not too great to admit it entirely within the field to be measured.

The total area of this coil is to be determined as accurately as possible at the time of winding.

It is generally better to pull this coil out of the field than to attempt to give it a motion of rotation. Sometimes this can be most conveniently accomplished by the aid of gravitation, the coil being attached to a weight, as in Fig. 38, and released by breaking the circuit which animates the small electromagnet (m).

Sometimes it is more convenient to make use of a spring, by means of which the coil may be removed from the field with the desired speed. In either case the interval of time should be comparable with that necessary to turn the earth inductor through 180°, otherwise the impedance of the circuit will not be quite the same for the two operations. A more important matter is that of the placing of the coil within the field. The distribution of lines of force in the fields of the character of those for the exploration of which the device under consideration is applied is by no means uniform. The number of lines which will penetrate the coil before it is taken from the field in successive trials,

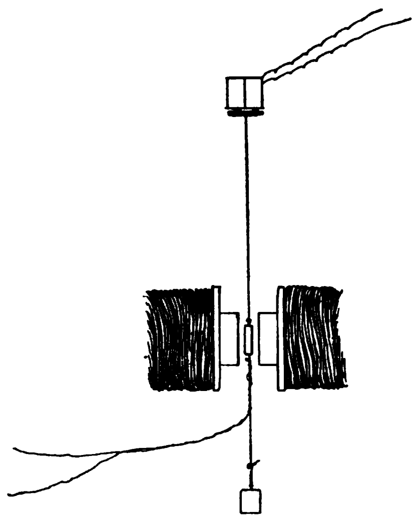

FIG. 38.

will often be found to vary considerably unless the greatest care is taken to bring it each time to precisely the same position.

In carrying out this method it will frequently be found necessary to vary the resistance R so as to render the deflections, due to the motion of the earth inductor and of the small coil, comparable. We have then in the computation of the ratio of the fields H_x and H_e, two areas (those of the respective coils) A_x and A_e, and two resistances R_x and R_e.

Equation 86 takes the following form when modified

to express the relations existing in this application of the method, viz.:

$$\frac{H_e}{H_x} = \frac{R_e\, A_x}{2\, R_x\, A_e} \frac{\sin \frac{1}{2}\, \vartheta_e}{\sin \frac{1}{2}\, \vartheta_x}. \tag{96}$$

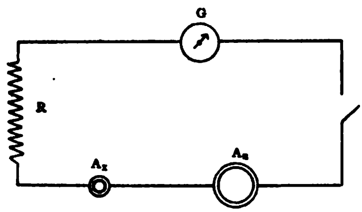

FIG. 39.

It is in most cases convenient to place the coils A_x and A_e in series with one another. The arrangement of connections is that shown in Fig. 39.

Where the comparison of fields is for the purpose of

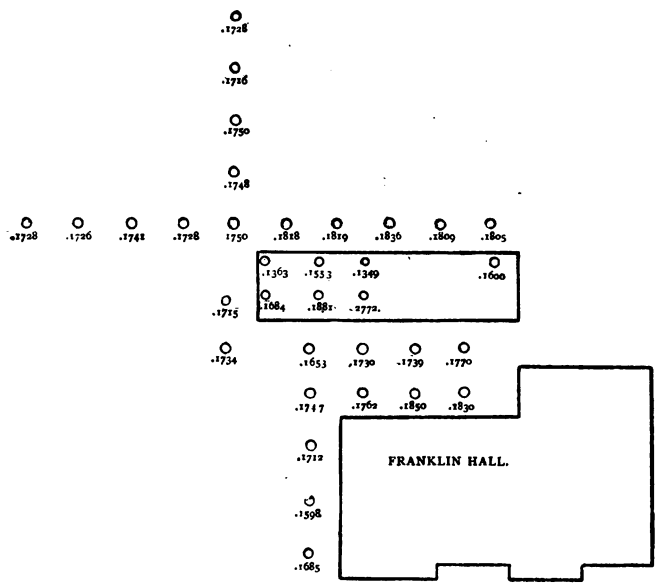

FIG. 40.

determining H in the locality where a galvanometer is placed it is important to make the measurement as closely as possible for the region occupied by the needle. It will not do to assume that the value of H throughout an ordinary laboratory room is nearly constant. R. W.

Wilson* has pointed out the wide range in the values of H within the limits of the Jefferson Laboratory at Cambridge, and his experience has been abundantly confirmed elsewhere. Fig. 40 shows the results of a similar survey recently made under the direction of the writer by Messrs. C. E. Hewitt and A. W. Smith. The exploration covered the interior and surroundings of the annex to Franklin Hall. The latter building is the physical laboratory of Cornell University, and the annex is a one-storied brick structure, 100 feet × 36 feet, situated a few feet to the north of the main laboratory. It contains but little iron aside from gas and steam pipes, some cast-iron wall brackets and some rods which serve to strengthen the roof.

In the accompanying map, Fig. 40, A is the annex, and the various small circles are stations at which H was determined. The values in C. G. S. units for each station are indicated. It will be seen that within the building H varied between .1363 and .2772, .1728 being the normal value in localities distant from local sources of disturbance.

It is interesting to note that the stations just north of the annex, also those along the north wall of Franklin Hall show values of H above the normal, while the row of stations just within the north wall all have low values. A similar survey of the neighborhood of the Magnetic Observatory of Cornell University, made by F. J. Rogers, shows the same phenomenon, and it seems probable that walls of masonry always exert a magnetic influence of the kind described.

* R. W. Wilson: *American Journal of Science*, vol. 39. p. 87. 1890.

LECTURE VI.

THE CONSTRUCTION OF GALVANOMETERS OF EXTREME SENSITIVENESS.

There is probably no instrument of precision used in physics at the present day which possesses so wide a range of sensitiveness as the galvanometer. The analytical balance which for a long time was the most remarkable of all instruments in this respect has fallen into second place on account of the remarkable developments as regards extreme sensitiveness which have been made in the construction of the galvanometer within a few years.

The equation of the galvanometer given in the first lecture indicates the lines along which increase in sensitiveness is to be attained. The constant of the gaivanometer is made up of two parts, H the horizontal component of the earth's magnetism, and G a factor which depends upon the dimensions of the coil and its distance from the needle.

It is evident from equation 7,

$$i = \frac{H}{G} \tan \vartheta,$$

that any method which will increase the value of G or diminish H will increase the sensitiveness of the instrument. The latter of these two processes has been discussed at some length in Lectures IV. and V., in the course of which it has been shown that an artifical field may be substituted for the earth's magnetic field, this field being either stronger or weaker than the earth's field according to the purpose for which the instrument is designed.

The final result of weakening the field around the magnet needle is to produce greater and greater instability of zero until finally the drifting of the needle becomes so rapid as to make it impossible to obtain readings. Thus a limit to the usefulness of the method of weakening the field for the purpose of increasing

the sensitiveness of the instrument is reached. In many operations, also, the lengthening of the period of vibration would in itself bring us to a limit of usefulness independent of the matter of magnetic drift.

Not less important than the reduction of H to small values is the increase of the quantity G in the constant of the galvanometer; and since this factor increases as the distance between the needle and the wire diminishes and increases also with the number of turns of wire in the coil, the problem of construction with view to extreme sensitiveness consists in part of reducing to a minimum the mean distance of the windings from the needle and of getting the largest number of complete turns for a given electrical resistance in the wire used in the construction of the instrument. A third, and very important, factor which enters into the consideration of the construction of galvanometers, is the lightness of the moving parts.

It is true that the sensitiveness of a galvanometer which is used following the method of permanent deflections is independent of the mass and moment of inertia of the moving parts, and independent of the magnetic moment of the needle also. In order to render a galvanometer, the suspended parts of which possess a large moment of inertia, as sensitive as one in which the moving parts are light, it is necessary, however, to increase the period of oscillation; and for many purposes this consideration taken by itself would dictate the reduction of the mass of the suspended portions to a minimum.

In nearly all operations of extreme sensitiveness, galvanometers are used ballistically, and under these conditions, both the moment of inertia and the magnetic moment are involved in the question of sensitiveness. The problem of the maker of such instruments, therefore, includes the question of securing as large a magnetic moment, and as small a moment of inertia as possible.

The sensitive galvanometer owes its origin to the demands of the student of radiant energy, and it was at the hands of Nobili and of Melloni that two of the important steps toward increased sensitiveness were made. The first of these was the introduction of the astatic pair in place of a single needle, a device which in modified and refined form holds its place in nearly all modern instruments. The other step consisted in the use of the telescope and mirror. The galvanometer is a direct descendant of the magnetic compass, and the user of the galvanometer inherited from his forerunner, the mariner who steered by the aid of the compass, the crude device of a metallic pointer moving over a divided

circle. The substitution of the angular movement of a ray of light, noted by the aid of the telescope and scale, was a great advance.

The next important step resulted from the demands of the needs of sub-marine telegraphy, and the requirements of this branch of applied electricity were completely and beautifully met in the mirror galvanometers of Thomson. In these well-known instruments the mass of the moving parts was for the first time reduced to a small quantity. The needle which, at the time of Melloni, was still really a needle, taken without any modification from the hands of the seamstress, and which in the later galvanometers of Siemens, Wiedemann, Edelmann and others, had undergone a series of transformations, none of which, however, had been in the direction of diminishing its mass materially, was reduced by Kelvin to a system of short, thin strips of steel, the length of each which was but a few millimeters, while the aggregate mass was a few milligrams.

In the hands of Kelvin, also, the mirror was reduced in weight in like proportion, by the substitution of microscopic cover glass for polished metal, or for the thick sheets of glass which had been used in the galvanometers of previous designers. In his instruments we find also, for the first time, the coil brought into really close proximity to the needles. The result of these changes was an instrument, the sensitiveness of which far exceeded that of any instruments which had previously existed, while the quickness of action necessary in cable signalling was secured by the reduction of the mass of the moving parts.

The discussion of the proper form and method of winding galvanometer coils to secure a maximum effect from a given weight or resistance of copper has been given by Maxwell in his treatise * The two most important points to be considered are the winding of the coil with different sizes of wire beginning with the smallest diameter, and the construction of the coil in such a manner as to bring the largest number of turns within a given effective distance from the needle.

If we consider the action upon a needle at *N* (Fig. 41) of a single turn of length *l*, causing a current *i*, we have for the strength of the component of the magnetic field at *N*, parallel to the axis of the galvanometer (see equation 3),

$$f_i = i\, l \frac{\sin \theta}{d^2}, \tag{97}$$

where *d* is the distance between the wire and the needle.

* Electricity and Magnetism, vol. ii., p. 360.

Since it is upon this field that the action of the galvanometer depends, it is clear that the problem consists in placing the winding, the radius of which is $d \sin \theta$, where it will make a field with the largest component in the required direction.

Maxwell has shown, in the paragraph just cited, that if a surface, the polar equation of which is

$$d^2 = x_1^2 \sin \theta. \tag{98}$$

be constructed, any circular winding of length l will produce a greater effect when it lies within the surface than when it lies outside it. It follows, therefore, that if a completed coil be of such shape that its surface is not of the above form, we may shift windings from

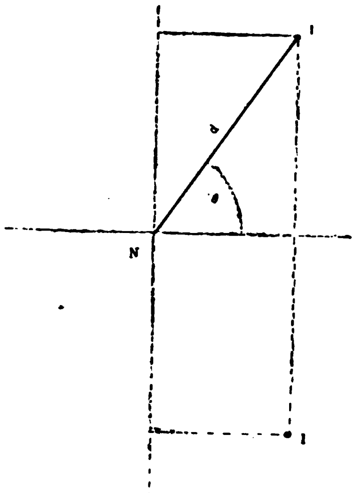

FIG. 41.

without the surface to a position within the same, thus improving its action without changing the amount of wire used. In a word, each layer of an ideal coil will always lie in a surface having an equation of the form of (98), and the value of x in the expression

$$x^2 = \frac{d^2}{\sin \theta} \tag{99}$$

will be constant for all its turns.

Fig. 42 is a diagram showing cross-sections of three such surfaces [Maxwell, ii., p. 361].

As regards the diameter of wire to be used in wind-

ing, the chief results of the discussion, cited above, are stated by Maxwell, as follows:

1. "If the method of covering the wire and of winding it is such that the space occupied by the metal bears the same proportion to the space between the wires whether the wire is thick or thin, then

$$\frac{Y}{y} \cdot \frac{d\,y}{d\,Y} = 1, \tag{100}$$

(where y is the radius of the wire and Y^2 is the area of the quadrilateral whose angles are the sections of the axes of four neighboring wires of the coil by a plane through the axis of the latter), and we must make both

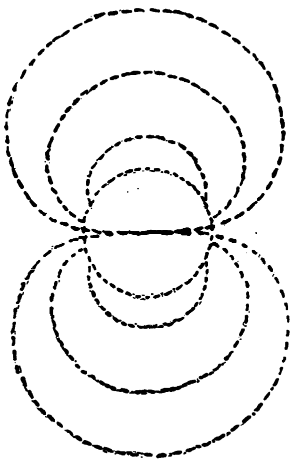

FIG. 42.

FIG. 43.

y and Y proportional to x (see equation 99); that is to say, *the diameter of the wire in any layer must be proportional to the linear dimension of that layer.*"

2. "If the thickness of the insulating covering is constant and equal to b, and if the wires are arranged in square order

$$Y = 2\,(y + b) \tag{101}$$

and the condition is

$$\frac{x^2\,(2\,y + b)}{y^3} = \text{constant}. \tag{102}$$

In this case the diameter of the wire increases with the diameter of the layer of which it forms a part, but not at so great a rate."

3. "If increase of resistance is not regarded as a defect, as when the external resistance is far greater than that of the galvanometer, or when our only object is to produce a field of intense force, we may make y and Y constant. In this case the value of G increases uniformly as the dimensions of the coil are increased so that there is no limit to the value of G except the labor and expense of making the coil."*

Aside from the questions of the shape of coils and the grading of the wire, the construction of a delicate galvanometer depends, as has been indicated already, upon the lightness of suspended parts, and the arrangement of same with reference to a minimum value of the moment of inertia, upon the strength of the needles, and upon the reduction of the space within which the suspended parts swing.

In all three of these particulars, modern practice seems to have been carried to a definite limit, beyond which it is difficult to proceed. The independent efforts of three or four of the most recent workers in this field have, indeed, led to the simultaneous development of instruments essentially identical and possessing very nearly the same relative figure of merit.

The use of miscroscope cover glass for the galvanometer mirror necessitates the careful study of the materials used, since glass in these thin layers is invariably badly warped. One plan has been to silver a very large number of covers, using Draper's solutions or the rather more convenient ones recommended by Kohlrausch. It is a matter of great difficulty to find among a lot of mirrors thus silvered, even one of any considerable size which presents a plane surface; but fortunately the reduction of the face of the mirror to a minimum is a desirable thing where we are seeking to construct an instrument with very small moment of inertia. One may rest content, therefore, with a few square millimeters of surface, provided by means of these the scale can be read.

Undoubtedly the best procedure is that recommended by Snow, Franklin and others, which consists in using a glass plate with a plane surface as a test plate and of laying down upon the same, one after another, the various pieces of cover glass from which mirrors are to be selected. If these be properly cleaned, interference bands will show themselves and from the shape of these it will be possible to determine whether any portion of the surface of each is approximately plane. Those which show the best surfaces are to be laid aside and silvered; the others are useless for the making of mir-

* Maxwell: Treatise ii., pp. 363–364.

rors. These selected glasses having been silvered in the usual manner, should then be cut into small rectangular pieces of the sizes desired.

The best size of mirror for galvanometers of the highest sensitiveness is the smallest size which will admit of readings being made with the telescope and scale. It is found that when such a mirror is cut to a width of less than two millimeters, diffraction fringes begin to disturb the image seriously. This, therefore, may be taken as the limiting size of such a mirror. We may gain something in surface without increasing the moment of inertia, appreciably by making the mirror oblong in shape and mounting it with its longer diameter parallel to the suspension rod. The best size for many purposes would seem to be a length of four to five, with a width from two to two and one-half millimeters.

Experience shows that one of the best materials for the suspension bar or rod upon which the elements of the astatic pair, together with the mirror, are to be mounted, is a slender fibre of glass. This must be as nearly straight as possible, since it forms the axis of rotation of the system. If a considerable number of glass fibers are made by drawing in the flame and are cut to the proper length, the straight ones may be selected by laying all upon a flat surface and rolling them both back and forth under the finger.

The question of the best size and shape for galvanometer needles, where the object is delicacy, is one upon which some further investigations should be made. At present it is generally conceded that a system of three or five small needles arranged side by side, instead of one heavier needle, gives a better result. Snow, in his galvanometer constructed in Berlin for the exploration of the bright line spectra of the metals, used six strips in each element of his astatic pair. These were arranged pair wise, back to back, one long between two shorter pairs. (Fig. 43.) Professor W. S. Franklin and the writer, in the course of a recent investigation requiring the very highest attainable sensitiveness in the galvanometer, made use of an instrument in which the needles were prepared as described below.*

"The elements of the astatic pair contained four magnets each. They were very nearly equal in strength, and were only two inches apart,—the mirror being placed below the coils instead of between them. By this arrangement the galvanometer was rendered comparatively insensible to magnetic disturbances. The galvanometer was provided with freshly made magnets

* See Physical Review, Vol. 1, p. 437.

just before being used,—a precaution which should always be taken in preparing for any important work requiring the last degree of sensitiveness, and care was taken to send no currents of any ordinary strength through the instrument. In the preparation of the magnets the following precautions were taken. Piano wire ¼ mm. in diameter was straightened by subjecting it to slight tension at a low red heat, and was cut into two-inch lengths. These were placed, two or three at a time, in an acute V-shaped iron trough, and after being heated uniformly to a cherry-red heat (800° C.) in a Bunsen flame, were quickly dropped into cool water. Two small pieces of exactly the same length were then cut from the central portion of each, and magnetized under similar conditions. The cutting was done by placing the hardened wire upon a smooth block of hard wood, and pressing an edged tool against it. If this procedure be carefully followed a highly astatic pair may always be obtained.

The two small magnets thus made from each piece were used, one in each of the elements of the astatic system. The galvanometer had a resistance of 15 ohms with its coils in series. When so arranged, and with a half-period of seven seconds, and scale distance of 120 cm., a deflection of 1 mm. corresponded to 6×10^{-10} amperes."

This refinement of the moving parts of the galvanometer would have been of little use but for the discovery of the remarkable qualities of quartz fibres made some years ago by Professor C. V. Boys. It has been abundantly shown by that physicist, and his statements have been verified by many others, that quartz possesses a strength beyond that of any other known material when drawn into fine threads or fibres, and that it is free from the structural defects of cocoon silk, which had been previously the best of known materials for the suspension of galvanometer needles.

The method of obtaining quartz fibres described by Boys is one the execution of which demands a considerable amount of manipulative skill and no little experience. The following is a more simple procedure.

The apparatus needed is a oxyhydrogen blowpipe, two pairs of crucible tongs and some bits of white quartz. The common variety of quartz crystal, known as milky quartz, serves well for this purpose. The materials should be crushed into small granules about three or four millimeters in diameter. These show a tendency to disintegration when first heated; when fused, however, the material goes over into a condition such that it may be placed in the flame over and over

again after becoming cold without further rupture.

The first step consists in making from these bits of pulverized quartz a number of short rods of the molten silica. These should be long enough so that they can be held in the flame, each end being within the jaws of a pair of tongs. When thus exposed to the hottest part of the gas jet, the middle softens readily and the rod can be drawn out into a fibre. These fibres are still much too heavy for use in the suspension of a delicate galvanometer, but they can be reduced to the desired fineness by the very simple process of holding them in the flame until they soften, when the draught of heated gas from the nozzle of the burner will be found sufficient to carry the softened fibre with it, drawing it out to a thinness which renders it suitable for the purpose now under consideration. With a little care these attenuated fibres, which are too small to be readily seen, can be secured, since they are attached at one end to the larger filament held in the hand. It is not always possible to secure the fibres in this way, many of them being torn loose and swept away in the currents of air. By placing at a safe distance above the flame a piece of canton flannel, however, these stray fibres will be driven against the rough surface of the cloth and will become entangled. In a very short time hundreds of them may be collected in this manner over the oxyhydrogen flame. Many of these will be of considerable length, and nearly all of them will be of sufficient fineness for use in galvanometers of the highest delicacy.

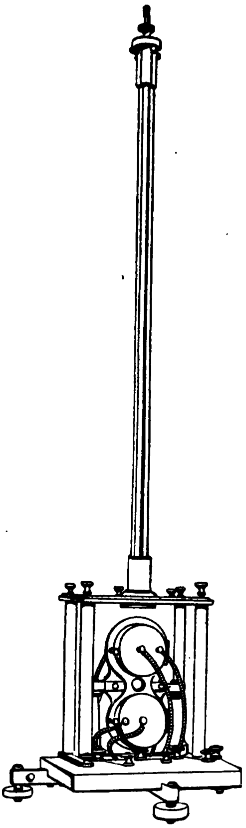

FIG. 44.

The advantage of the astatic pair in the construction of galvanometers having been once recognized, it was a very natural extension of the principle to introduce a second set of coils, so that each element of the pair might be brought into a stronger field due to the current. Kelvin made use of such an arrangement in one of his types of galvanometer, bringing the mirror to a

position midway between the two groups of needles, a procedure which has been widely followed by others. Fig. 44 shows an instrument in which four coils are used with the mirror placed between them as above described. Fig. 45 shows the same instrument on a larger scale with the short coils swung away so as to show the interior. Fig. 46 shows the arrangement of

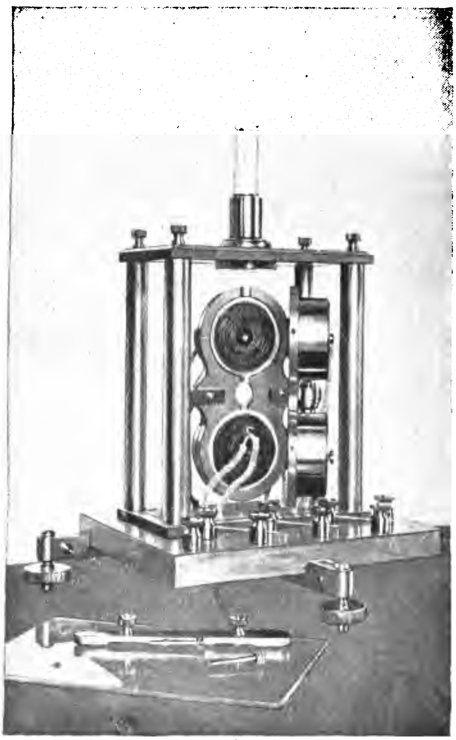

FIG. 45.

the needles and mirror in this instrument about life size.

The suspended parts of such galvanometers having been reduced to a minimum as regards mass and moment of inertia, it was a natural mistake to suppose that even the quartz fibre must be of great length in order that its moment of torsion should remain inappreciable. Thus in the galvanometer just depicted a fibre half a

meter long was used, and of many other galvanometers made at that time the same thing is true. While it is easy to obtain quartz fibres of the requisite length and fineness, it is a much more serious matter to mount a long fibre than a short one, and after the instrument has been successfully set up, the question of keeping it adjusted, as to level, so that the suspended parts shall be free within the very narrow space allotted to them, becomes a difficult one. Subsequent experience has shown that a fibre five to ten centimeters long is sufficient, even in the case of the lightest galvanometers.

There are certain advantages in bringing the elements

FIG. 46.

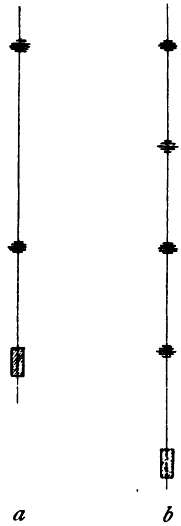

FIG. 47.

of an astatic pair near to one another, and to attain this end, the mirror is sometimes placed at the bottom of the suspension rods as shown in Fig. 47 (*a*) which gives the arrangement of mirror and needles in such a galvanometer. Fig. 48 shows an instrument of this type, constructed as were also the galvanometers shown in Fig. 44 and 49, by F. C. Fowler, instrument maker to the Department of Physics in Cornell University. This instrument has four coils placed pair-wise one above another and adjustable as to the distance between them. A somewhat similar instrument with eight coils and four sets of needles mounted in four equally distant

groups with a mirror at the bottom is shown in Fig. 49. A diagram of the suspended parts is given in Fig. 47 (*b*) Figures 50, 51 and 52 give some details of the galvanometer shown in Fig. 49.

The only factor in the construction of a sensitive galvanometer, which we have still to consider, is that of the distance between the opposite coils. With mirrors two millimeters across and needles three or four millimeters long, it would seem that the clearance necessary

FIG. 48.

to give freedom of action might be extremely small. In this regard, however, a limit is soon reached on account of the difficulties which arise through the magnetic properties of the materials of which the coils themselves are constructed. If the layers of wire lying next the needle be insulated with the usual green silk, it will be found that these when brought near the suspended needles will attract the same strongly and will

soon interfere altogether with the requisite freedom of motion. The substitution of white for the green covered wire seems to mitigate this trouble to some degree, but it is a matter of great difficulty to find thoroughly non-magnetic insulated copper wire. Then, again, in the process of handling the instrument for mounting, the silk covering, and the shellaced surfaces, tend to become electrified and much annoyance arises from this source. It is, indeed, sometimes necessary to cover the

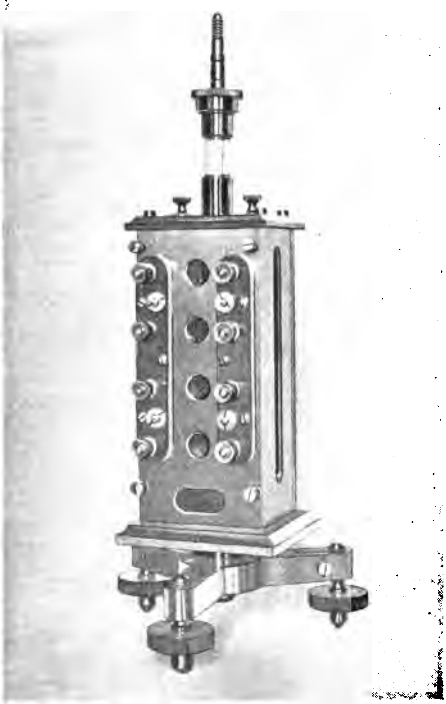

FIG. 49.

entire inner face of the coils with gold leaf and to ground the same before the needles can be made to swing freely in close proximity to the coils. The writer has never found it practicable to work with galvanometers in which the average clearance space was reduced to less than one millimeter. Fig. 53, which shows the position of the coils of Snow's galvanometer, indicates that in his instrument, the clearance was approximately that just mentioned.

As regards the sensitiveness of galvanometers constructed in accordance with the principles laid down in this lecture, the following data may be of interest:

B. W. Snow, 1892, constructed the galvanometer, the coils of which are shown in Fig. 53, and the suspended parts in Fig. 43. He obtained a figure of merit 1.5×10^{-11} amperes, with a vibration period of 20 seconds and a scale 300 cm. from the instrument. The above applies to a deflection of one millimeter. The resistance of the

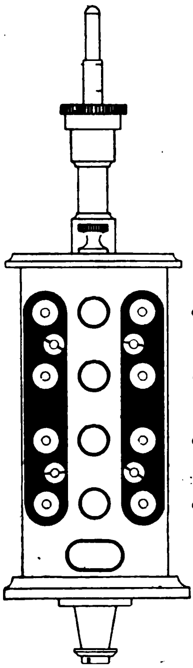

FIG. 50.

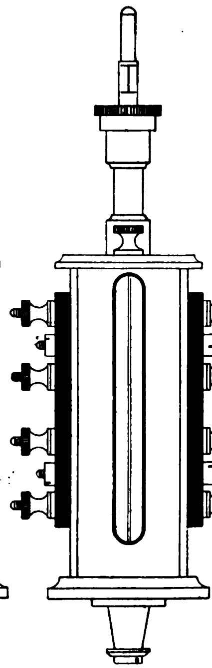

FIG. 51.

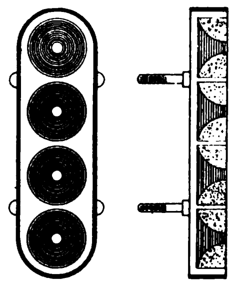

FIG. 52.

instrument was 140 ohms. In the same year E. F. Nichols and the writer used the galvanometer with long suspension fibre, shown in Fig. 44. The aggregate weight of the moving parts of this instrument was 48 mg. With the coils in multiple the resistance was 9.3 ohms. The sensitiveness reached 1×10^{-10} amperes with the coils thus connected. The scale was about 150 cm. from the instrument, and the period about 10 seconds. W. S. Franklin and the writer found for the

galvanometer used in their study of the condition of the ether surrounding a moving body, the method of constructing the magnets of which instrument has just been described, a figure of merit of 6×10^{-10} amperes with a resistance of 150 ohms, a period of 7 seconds and a scale of 120 cm. distant. In this case also the figure of merit refers to a millimeter of deflection.*

Paschen† also, who constructed a special galvanometer for the exploration of the very weak spectra afforded by the diffraction grating, obtained $1\ 6 \times 10^{-11}$ amperes for one millimeter with 20 ohms, a period of 30 seconds and a scale 270 cm. distant.

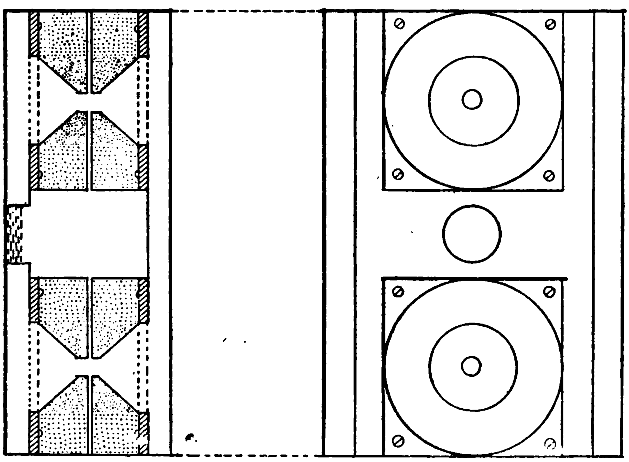

FIG. 53

* For these three cases see *Physical Review*, vol. i.
† Paschen: *Wiedemann's Annalen*, 48, p. 284.

LECTURE VII.

Special Applications of the Galvanometer to the Measurement of Current and Resistance.

1. *Measurement of feeble currents.*—One of the important uses of the galvanometer is for the detection and measurement of currents of exceedingly small intensity, for which purpose the instrument must be especially adapted by constructing it with reference to the reduction of the constant to the smallest possible value. This reduction, as has already been pointed out (Lecture VI.), is attained by bringing the wire as near as possible to the needle, by reducing the moment of inertia of the moving parts to a minimum, by making use of the astatic system and by the artificial reduction of the magnetic field within which the needle swings to a very small intensity.

Since the constant of such instruments cannot be determined by computation from the dimensions of the coils the galvanometer must, be calibrated. In addition to the absolute calibration, means must be devised for repeated re-determinations of the fluctuations to which the figure ot merit of galvanometers of extreme delicacy are subject.

The calibration may be made:

(1.) By the use of the Clark's cell, the greatest care being taken to fulfill the conditions under which this form of cell affords reliable results. A detailed description of this cell and of the method of using it will be given in the Lecture VIII.

(2.) The galvanometer may be calibrated also by placing it in shunt around a known resistance (see Fig. 54). This method involves the use of a second galvanometer, G, of known constant, by means of which the current flowing through the resistance in question can be measured. The size of this resistance, R_s, will de-

pend upon the figure of merit of the instrument. It must be of such size that when a measurable current traverses it, the difference of potential between its terminals, shall give a suitable deflection to the galvanometer to be tested. In case of instruments of the highest delicacy this involves the reduction of the current to values too small to be measured upon a stand-

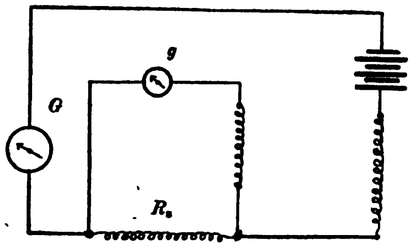

FIG. 54.

ard instrument, or the increase of the resistance to such an extent as to render accurate knowledge of its value, a matter of difficulty. In such cases it is better to use a multiple shunt, the arrangement of which is shown in Fig. 55.

This device makes it possible to standardize instruments of extreme sensitiveness with a fair degree of accuracy. It is true that increased error is introduced by the use of the second shunt, but it is also true that

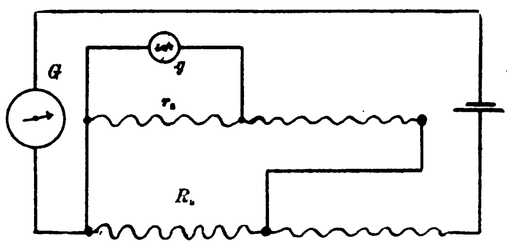

FIG. 55.

very precise determinations of the figure of merit of galvanometers of the highest delicacy are rendered useless from the fact that such galvanometers are subject to continual changes of constant. Almost in proportion as the sensitiveness of the instrument rises beyond a certain value, the possibility of precise determination of its sensitiveness diminishes.

Arrangements for the calibration of the instrument having been completed, it is necessary to provide some means of keeping pace with the fluctuations of constant already referred to. Probably the very best means for this purpose is a subsidiary coil placed at a suitable distance behind the galvanometer. This coil will act upon the needle, but the current through it necessary to produce a deflection will be very much larger than that which would give the same result when sent through the coils of the galvanometer itself. The number of turns of this subsidiary coil may be few, but its distance from the needle is necessarily very considerable.

The subsidiary coil should be placed in shunt circuit

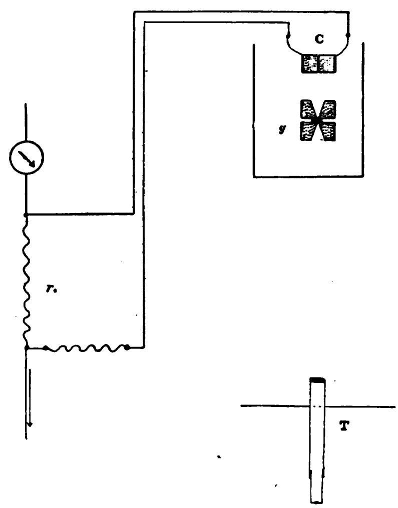

FIG. 56.

with a compensated resistance, r_s, Fig. 56; through which a known current flows. Upon closing this subsidiary circuit a deflection will be produced by the proper adjustment of the resistance in the circuit. The deflection may be brought to size approximately equal to that of the deflections which the galvanometer will give in the operations to which it is to be subjected by further adjustment of the resistance r_c. Immediately after the completion of the absolute calibration, the deflection due to the subsidiary coil with known resistance flowing through the compensated resistance should be noted, and this deflection be made to serve as a refer-

ence factor in all subsequent operations. In order to keep track of changes in the constant of the galvanometer occurring from time to time thereafter, it will only be necessary to send the same current through the subsidiary coil as that which was sent through it at the time of the calibration. The range in the deflections thus produced will then represent the range of the figure of merit of the galvanometer. The arrangement of the subsidiary coil and its circuit is shown in Fig. 56, to which reference has just been made.

Another device for obtaining the same end, which has the advantage of not requiring the use of a standard instrument or ammeter, is as follows:

A thermo-element consisting of an iron or German silver wire about one meter long, the ends of which are

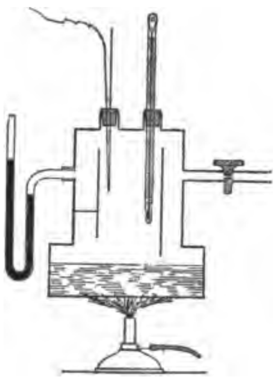

FIG. 57.

soldered to copper wires, is constructed. One junction of this thermo-element is packed in ice, the other is placed in a steam bath, the pressure of which can be regulated so as to make a delicate thermometer situated in the same bath read constantly 100 degrees. Such a thermo-element will give a small but perfectly constant electromotive force so long as a constant difference of temperature between its terminals is maintained. This thermo-element may be used two ways: (1) to produce deflections by placing it in circuit with the galvanometer coils themselves from time to time (see Fig. 58); (2) in the case of galvanometers of extreme sensitiveness, by placing the thermo-element in circuit with the subsidiary coil.

In all cases in which it is necessary to reduce the galvanometer to its condition of maximum sensitiveness, a

point is reached at which the drift of the zero due to magnetic disturbances introduces serious error. It has already been shown in the lecture on galvanometers with artificial fields, that in the case of instruments with weak fields every fluctuation in strength of the earth's magnetic forces has an exaggerated effect upon the needle. It is, indeed, oftentimes impossible to maintain a sensitive galvanometer with approximately fixed zero long enough to obtain a permanent deflection.

In all such cases the only remedy is to make use of the ballistic method in which the zero of the galvanometer at the instant before closing circuit is noted, and the circuit is closed during a single swing of the instrument. It is then opened, the reading at the end of this

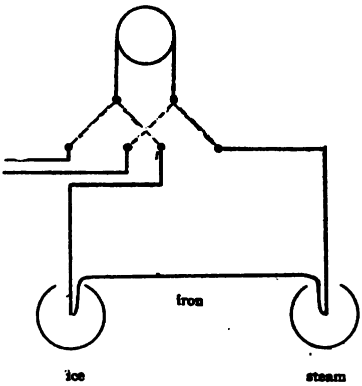

FIG. 58.

swing is noted, and finally the point which the needle reaches in its first return swing is observed. If the average between the reading of the return swing and the original zero be taken as a corrected zero, the drift of the galvanometer in the short intervening interval will be almost entirely eliminated. Deflections computed in this way should be interpreted by means of a calibration performed in a corresponding manner instead of a calibration by permanent deflections.

In some extreme cases, such as occur in work with the thermo-pile or bolometer, it is found to be impossible to open and close a switch in the galvanometer circuit at all without producing disturbances of great magnitude. In these cases fortunately, it will be found possible to use the galvanometer in a permanently

closed circuit, the deflection being produced by the exposure of the thermo-pile or bolometer, to the source of radiation under observation, by the sudden removal of an intervening screen.

The operation consists in reading the instrument with the circuit closed, and with the screen intervening between the source of light and the thermo-pile or bolometer. This gives the zero point of the deflection. The screen is then removed during one swing of the galvanometer and is restored to its place. The extreme reading of the swing is read, also the return swing, and the deflection is computed from these three observations as already indicated. Theory would lead us to expect that the deflection thus obtained would be proportional

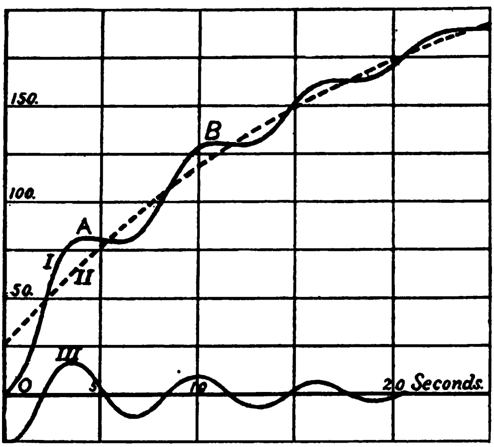

FIG. 59.

to the permanent deflection of a galvanometer, the zero of which was fixed. Professor Ernest Merritt* has shown experimentally that these conditions are fulfilled in practice. Fig. 59 gives a curve obtained by him from observation of a galvanometer used in this manner on closed circuit, the readings being carried on not only during the first swing, but for a much longer period, up to the time, indeed, when the needle reached its final position.

In this figure curve I gives the observed movement of the needle, the galvanometer being in closed circuit with a thermo pile which was suddenly exposed to heat at the time, marked *O* seconds. The analysis of this

* Merritt; *American Journal of Science*, vol. xli., p. 417.

curve shows it to be the resultant of curves II and III, the latter, of which is a trace of logarithmic decrements the period of which agrees with the free swing of the needle.

Merritt found that in all cases the first throw of the galganometer needle was proportional to its permanent deflection.

2. *Measurement of heavy currents.*—Another important problem in the measurement of currents is that where the strength of the current is too great for direct determinations. Under these circumstances the method by fall of potential is to be preferred. The only difficulty of carrying out this method successfully lies in the maintenance of a constant temperature in the resist-

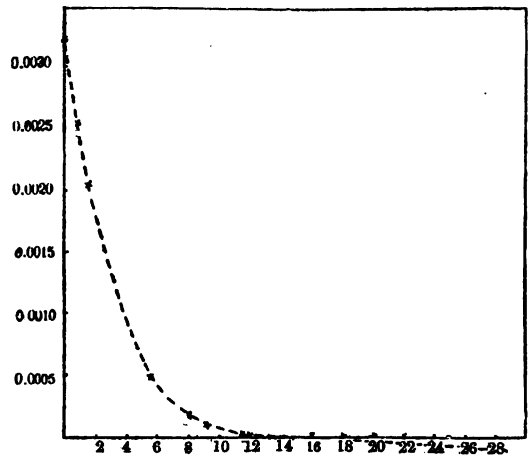

FIG. 60.

ance around which the galvanometer is shunted, but this difficulty has been found so great as to lead to the condemnation of the method on the part of many investigators. Two remedies have been proposed: (1) the use of a material for the shunt resistance which does not vary with the temperature. Certain alloys of manganese and copper, or manganese, nickel and copper, are known to possess this property, but their other properties have not been studied as yet with sufficient thoroughness to warrant unreserved confidence in their stability. In a word, we do not know whether shunts constructed of such material will remain of constant resistance when subjected to the action of heavy currents.

From the measurements made upon this class of alloys, however, it appears that unless very particular attention is paid to the matter of annealing, etc., changes of specific resistances are sure to occur as the result of every subsequent fluctuation of temperature. B. H. Blood* in the course of an examination of an alloy containing .8082 copper and .1912 ferro-manganese, found that eight successive heatings to 100° C. with alternate coolings to 20° C., gave the results indicated in the following table.

EFFECT OF REPEATED HEATING AND COOLING UPON THE RESISTANCE OF AN ALLOY OF FERRO-MANGANESE AND COPPER (HARD DRAWN).

Observation.	Temperature.	Specific Resistance.	Relative Resistance.
	Degrees.		
1	20	30.380	1.0000
2	100	30.186	.99331
3	20	30.163	.99287
4	100	30.151	.99255
5	20	30.138	.99202
6	100	30.121	.99180
7	20	30.118	.99134
8	100	30.118	.99134
9	20	30.105	.99093
10	100	30.099	.99072
11	20	30.092	.99051
12	100	30.104	.99092
13	20	30.079	.99007
14	100	30.104	.99092
15	20	30.072	.98985

As regards temperature coefficients of resistance, however, provided a method of treatment can be found to check the behavior just referred to, this class of alloys leaves little to be desired. The coefficient, indeed, seems to depend directly upon of the percentage of ferromanganese present, and to pass from positive to negative values when 18 per cent. of that material is combined with the copper. This is shown in the curve (Fig. 60) platted from results obtained by the same observer. In this diagram ordinates are coefficients and abscissæ are percentages of ferro-manganese.

Since the currents, the measurement of which we are now considering, are too heavy to allow the use of the compensated resistance of copper and carbon, which is to be described in Lecture VIII., and since the only known available material which is without a coefficient is questioned, recourse must be had to a metallic shunt, and to some device by means of which its temperature can be controlled. This, indeed, is the plan prescribed

* *American Journal of Science*, vol. xxxix., p. 473.

LECTURE VIII.

The Measurement of Electromotive Force by Means of the Galvanometer.

The Determination of the Electromotive Force in Absolute Measure.—The best instrument for this practice is a tangent galvanometer of considerable sensitiveness, the constant of which has been determined by measurement of the coils. The resistance of the galvanometer itself should be as low as possible, and it should be used in series with a variable resistance, the absolute values of all the coils of which are accurately known. Fig. 62 shows the arrangement of this apparatus for the determination of the difference of potential between points *a* and *b* in any circuit.

The method is capable of very wide range. The upper limit is reached when the electromotive force to be measured is so great that the resistance R_p cannot be made large enough to reduce the deflection to a readable size. The lower limit is reached when R_p becomes zero and the deflections are too small for accuracy. The method may be extended in the direction of high electro-motive forces by the use of a galvanometer with two coils capable of being placed in series, or in multiple, or of being used differentially.

Fig. 63 is from a photograph of a small tangent galvanometer designed especially for this purpose by Prof. W. A. Anthony. The windings are arranged in four parts which may be thrown together in either of the three ways indicated above, by the use of a small divided block with plugs placed upon the base of the instrument. The range of usefulness is very wide, extending from a thousandth of a volt per centimeter deflection with no resistance in circuit, to over twenty volts per centimeter deflection, when used in series with a megohm box. Used differentially, the instrument extends to quite as high potentials as one could expect to measure by such a method.

In a case of very low electromotive forces, the standard galvanometer must be replaced by one of high sensitiveness, which it is necessary to calibrate before using. Probably the best method of calibrating such a galvanometer consists in looping the same with suitable resistance in circuit around a compensated resistance through which a known current flows. (See Fig. 64.) With a suitable standard instrument, A, for the measurement of current flowing through this compensated resistance and with adjustable resistance R_s and R_g in the main circuit and in the circuit of the galvanometer to be tested, the latter can be accurately calibrated throughout its entire range.

The compensated resistance coil suitable for this purpose, is made of copper, which, as is well known, possesses a positive temperature coefficient of about 0.004 placed in multiple with a rod of graphitic carbon, the temperature coefficient of which is always negative with a value varying considerably, but always much smaller than that of copper. (See Fig. 65.) Some data concerning the variations of the temperature coefficient for resistance to be met with in testing various kinds of carbon are given in the following table.

INFLUENCE OF TEMPERATURE UPON THE RESISTANCE OF VARIOUS VARIETIES OF CARBON.

Observer.	Variety of Carbon.	Temperature Interval	Mean coefficient per deg.
Werner Siemens (*Wiedemann's Annalen*, 10, p. 560)	Gas carbon	0 to 200°	.000345
	Pressed gas carbon	0 to 200°	.000301
Borgmann (*Journal der Russischen Physikalischen Gesellschaft*, 9, p. 163.)	Charcoal	26 to 260°	.00370
	Anthracite	20 to 250°	.00265
	Graphite	25 to 250°	.00082
	Coke	26 to 245°	.00026
Kemlein (*Wiedemann's Annalen*, 12, p. 73.)	Gas carbon (coarse)	18 to 200°	.000285
	Gas carbon (fine)	18 to 200°	.000287
	Carré's carbons	18 to 200°	.000321
Muroaka (*Wiedemann's Annalen*, 13, p. 307.)	Paris retort carbons		.000300
	Arc light carbons		.000405
	Arc light carbons		.000425
	Gaudoin's carbons		.000415
	Kaiser and Schmidt's carbons		.000370
	Heilmann's carbons		.000240
	Arc light carbons		.000156
	Siberian graphite		.000739
	Faber's lead pencil graphite		.000588

For the range of temperatures through which it is desired that this resistance shall be constant, we may assume that the coefficients are both constant quantities.

The problem to be solved in the construction of a

compensated coil for the range of temperatures in question consists in determining the proper amount of carbon and of copper to be placed in multiple circuit so that the total resistance of the combination shall not vary. This condition will be met, provided the rise in resistance on the part of the copper is exactly counterbalanced by the increased conductivity of the carbon. These conditions will be approximately fulfilled when

$$R_m'\left(\frac{a_c\, t}{1-a_c\, t}\right) = R_c'\left(\frac{a_m\, t}{1+a_m\, t}\right), \tag{103}$$

in which equation R_m' is the resistance of copper, R_c' that of the carbon at a given temperature, (say 20°C), while a_m and a_c are the respective coefficients and t is a temperature (say 100°C), up to which compensation is desired.

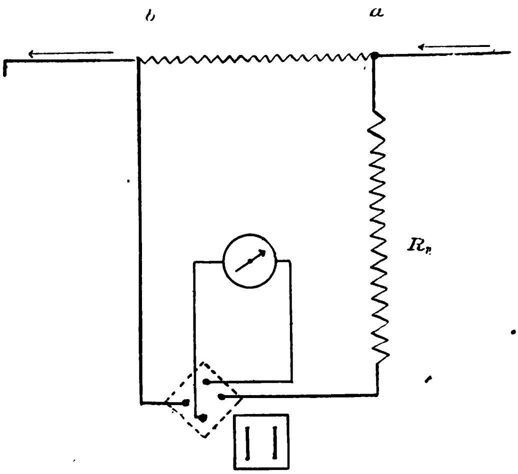

FIG. 62.

Assuming that the combination would be subject to the law of parallel circuits, we have for the total resistance R:

$$R=\frac{R_m\, R_c}{R_m+R_c}, \tag{104}$$

where R_m is the resistance of the metal and R_c the resistance of the carbon.

The equation of condition, for complete compensation will be

$$\frac{R_m'\, R_c'}{R_m'+R_c'}=\frac{R_m''\, R_c''}{R_m''+R_c''}, \tag{105}$$

where R_m' R_c' are the resistances of the components and

R_m'' R_c'', the corresponding resistances at any other temperature within the limits of temperature for which compensation exists.

Now the variation of the resistance of a metal with the temperature may be expressed by an equation of the form:

$$R_m'' = R_m' \, (1 + at \pm bt^2), \qquad (106)$$

where a and b are coefficients to be determined by experiment.

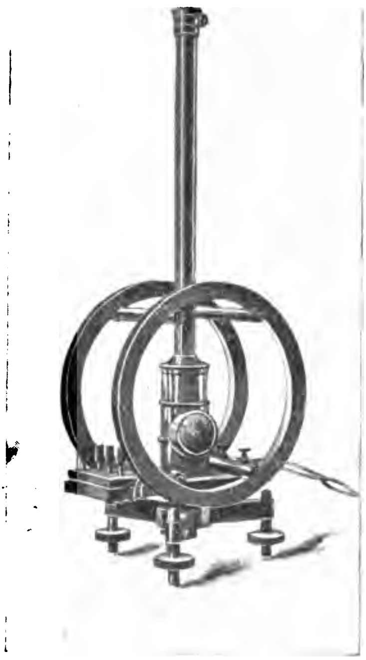

FIG. 63.

In the case of carbon, the coefficient a will have a negative value, and the equation will take the following form:

$$R_c'' = R_c' \, (1 - at \pm bt^2). \qquad (107)$$

Between 0° and 100° the value of b in the case both of copper and carbon is very small. A determination of the coefficients for copper, made in the Physical

Laboratory of Cornell University, for example, yielded the equation :

$$R_m'' = R_m' (1 + .00380\, t + .00000047\, t^2). \quad (108)$$

The experiments, which covered a range of 100°, are in close agreement with the results published by Matthiessen.

In the above equation, for $t = 100°$, we have $bt^2 = .0047$. If we neglect the coefficient b and adopt for a the mean coefficient between 0° and 100°, we may write the equation in the simpler form,

$$R_m'' = R_m' (1 + .003847\, t), \quad (109)$$

which will give results agreeing with the complete form at 0° and 100° and will have a maximum error, at 50°, of .0008.

In the case of carbon the coefficient b may also be neglected without appreciable error.

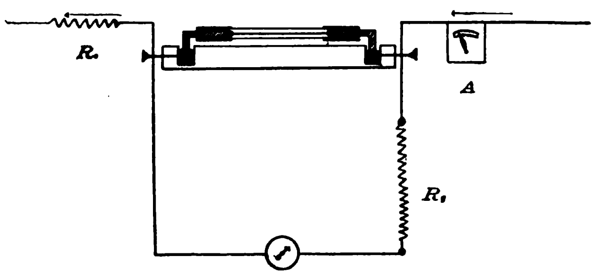

FIG. 64.

A Carrè pencil, measured in the same laboratory, gave as mean coefficient between 0° and 100°, the value .000235.

We may, therefore, write the equation for this variety of carbon as follows :

$$R_c'' = R_c' (1 - .000235\, t). \quad (110)$$

In combining copper and carbon in such proportions that the resulting resistance shall be independent of the temperature, the equation of condition must be satisfied.

This equation will be satisfied only for a range of temperatures throughout, which b^2t is negligible, in the case of both substances.

For such a range of temperatures we have, however, as already indicated,

$$\begin{aligned} R_m'' &= R_m' (1 + a_m\, t) \\ R_c'' &= R_c' (1 - a_c\, t); \end{aligned} \quad (111)$$

when a_c and a_m are the coefficients for carbon and copper respectively.

Within such limits

$$\frac{R_m' \, R_c'}{R_m' + R_c'} = \frac{R_m' \, (1 + a_m \, t) \, R_c' \, (1 - a_c \, t)}{R_m' \, (1 + a_m \, t) + R_c' \, (1 - a_c \, t)}; \quad (112)$$

which is readily reducible with a sufficient degree of approximation to the form given in (103).

A convenient form for such a resistance is shown in Fig. 66. It consists of a rod of carbon about twenty centimeters in length, the ends of which have been copper plated and then soldered to massive copper terminals. These are bent at right angles and amalgamated for convenience in making connections by means of mercury cups. (See Fig. 65.) The copper compen-

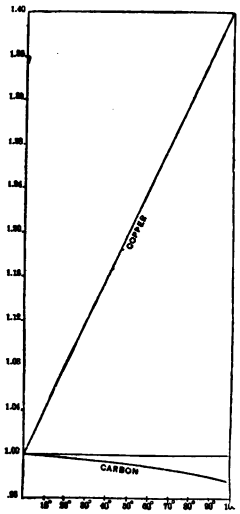

FIG. 65.

sation may be obtained by means of a insulated wire of proper resistance, coiled snugly in spiral form around the rod from end to end. A glass tube protects the apparatus from damage.

Compensated resistances can be made, in a variety of other forms, according to the materials in hand and the requirements of the case. A very simple and excellent form consists of an incandescent lamp with german silver wire placed in series or in multiple with the filament. This form is of much higher resistance than that depicted in Fig. 66, and will carry less current. It is available, indeed, only in cases where the heating effect of the current would be inappreciable.

The use of a compensated resistance, of the character first described, is very convenient, since it does away with the necessity of considering fluctuations to which an uncompensated conductor will be subject as the result of the heating effect of the current flowing in it, and of temperature disturbances from without. In Lecture VII, it has been shown, however, that it is entirely practicable to determine accurately the resistance of a metallic conductor when carrying current. By the application of the methods therein described, especially of the second method of the time curve, entirely satisfactory calibrations of a sensitive galvanometer for the measurement of low electromotive forces may be obtained.

An important example of the use of sensitive galvanometers in the determination of electromotive force is found in the comparison of standard cells. The galvanometer for such purposes should be sensitive to one hundred-thousandth of a volt, and its figure of merit should be known with a fair degree of accuracy. The conditions of such determinations involve the ready measurement of small differences of potential with the

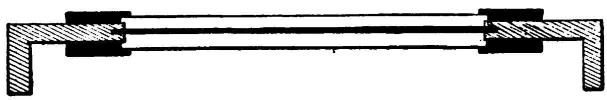

FIG. 66.

expenditure of as little current as possible. The usual method of proceedure is to place the two cells which are to be compared in opposition to each other, and to use the galvanometer ballistically, a sufficient amount of resistance being placed in circuit to place its deflection to a suitable quantity.* One of these two cells will usually be the standard with which others are to be compared. The standard cell must be maintained at a constant temperature, viz., that for which its electromotive force has been previously determined. By substituting successively the various cells which are to be compared, these being placed in such a direction as to oppose the standard, very accurate determinations of the ratios of their electromotive forces to that of the standard may be obtained. In the case of the Clark cell the temperature coefficient has been made a matter of careful study by Clark, Rayleigh, Wright, Helmholtz, Kittler, and more recently by Carhart. The value of this coefficient, according to the different observers,

* For details of methods of testing, see Carhart; Primary Batteries.

varies through a considerable range, as will appear from the following table:

Observer.	Loss per degree Centigrade.
Clark[1]	0.0006
Helmholtz[2]	0.0008
Kittler[3]	0.0008
Rayleigh[4]	0.00077
Wright[5]	0.00041
Von Ettingshausen[6]	0.00068
Carhart[7]	0.000387

The construction of Rayleigh's form of the Clark cell is shown in Fig. 67. Such cells give constant and comparable values only under very careful treatment, and

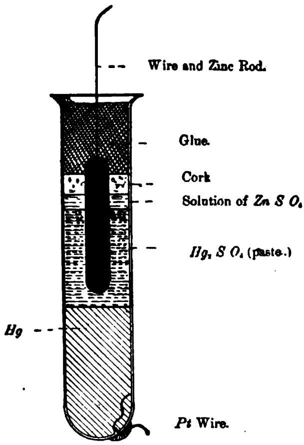

FIG. 67.

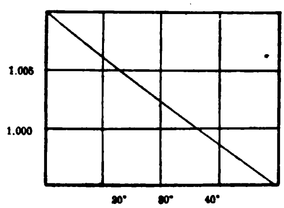

FIG. 68.

even when handled in a manner which ensures their integrity, they possess a high temperature coefficient.

The source of the variation in the temperature cofficient of Clark cells, has been clearly pointed out by Carhart, who has succeeded by modifying the cell in such a way as to reduce the coefficient to a minimum value, by eliminating one of its elements, the variation in the concentration of the solution of zinc sulphate.

This done, by the use of a solution which is saturated at 0° (or at some temperature below that at which the cell is to be maintained) the variation due to changes in

1. Latimer Clark; Journal of the Society of Telegraph Engineers, Vol. 7, p. 53.
2. Von Helmholtz; Sitzungs Berichte der Berliner Akademi, 1882, p. 26.
3. Kittler; Wiedemann's Annalen, 17, p. 890.
4. Rayleigh and Sidgwick; Proceedings, Royal Society, 17, 1884.
5. Wright; Philos Magazine, 5, Vol. 16, p. 25, 1883.
6. Von Ettingshausen; Zeitschrift fur Electrotechniker, (Wien), 1884, p. 1.
7. Carhart; Primary Batteries, p. 93.

the density of this electrolyte vanishes and the temperature coefficient falls to its normal value (0.000387). The coefficient of such cells can be expressed graphically, as function of the temperature, as in Fig. 68 or by the equation,

$$E_t = E_{15}(1 - 0.000387(t - 15) + 0.0000005(t - 15)^2 \qquad (113)$$

The Clark cell was made use of by the Chamber of Delegates of the Chicago International Congress of Electricians in the establishment of a practical unit of electromotive force. Their definition was as follow :

"The *International volt* is the electromotive force which steadily applied to a conductor whose resistance is one international ohm, will produce a current of one international ampere, and which is represented sufficiently well for practical use by $\frac{1000}{1434}$ of the electromotive force between the poles of electrodes of the voltaic cell, known as Clark's cell, at a temperature of 15°."

If one is in posession of two Clark cells, the temperature coefficients of which are well known, these may be used in combination as a means of procuring a standard of very small electromotive force. By maintaining the two at slightly different temperatures; and placing them in circuit, back to back, they may be made to produce a perfectly definite and very small electromotive force by their differential action, and in this combination may be used for the purpose of calibration. A much more convenient source of small electromotive force which serves as an admirable secondary standard, is a thermo-element of copper-iron, copper-german silver or platinum-iridium, according to the range desired.

It is not possible, in constructing such a thermo-element out of the materials ordinarily attainable, to secure a standard of electromotive force which is absolute in the sense of giving the same values in a case of different individual elements, since the differences between the various members of a series of such elements, even when these are constructed as nearly as possible from like materials, will be found to be considerable. Once constructed, however, such thermo-elements are not subject to marked fluctuations in their character. It is possible, therefore, to make a thermo-element and to calibrate it once for all. It may then be used as a secondary standard of small electromotive forces; the only further precautions being those involved in bringing the two junctions to a known temperature difference and maintaining them there. The temperatures most easily maintained are, of course, those of melted ice and of steam at normal pressure. The arrangement of such a standard is described in a previous lecture.

LECTURE IX.

The Use of the Galvanometer for the Measurement of Temperature.

The types of instrument useful for the determination of temperatures are those of maximum sensitiveness, described in Lecture VI. In thermometric work the galvanometer is used :

(1) In measuring changes in the resistance of a wire by the method of fall of potential.

(2) In the Wheatstone bridge.

(3) In circuit with a thermo-pile or thermo-electric couple.

The first method has a very wide range of usefulness. If, for example, temperatures are to be determined in a locality in which the mercury thermometer cannot be used, a coil of pure copper wire may be prepared, the resistance of which at a known temperature is accurately determined once for all, as also its temperature coefficient for the entire range of temperatures under consideration. This coil having been placed in the locality for which the temperatures are to be measured is connected with the galvanometer by line wires of negligible resistance. A comparison coil, the resistance of which should be as nearly as is convenient the same as that of the temperature coil, is placed in a bath of constant temperature. This comparison coil should be constructed of material having as small a coefficient temperature as possible, or it may be compensated by the methods described in Lecture VIII. The temperature coil, R_t, and the comparison coil, R_c, are placed in series with some suitable source of current (B), and the circuit is permanently closed. The galvanometer is placed alternately in shunt with the two coils (see Fig. 69) and the ratio of the deflections thus obtained gives the resistance of the temperature coil in terms of that of the other. As has already been pointed out in the lecture just referred to, this method of alter-

nate deflections eliminates the fluctuations in the figure of merit in the galvanometer and affords a means of ready and accurate measurement of the variations of resistance of the temperature coil and so indirectly of the changes of temperature which occur in the locality in which it is placed. When it is desired to integrate the temperatures existing throughout a region of considerable extent, the wire instead of being wound into a coil is carried through the entire region to be studied. If, for example, we desire to know the average temperature of a standard bar, the length of which is to be measured, the wire may be wound around the bar longitudinally a sufficient number of turns being made to give the desired resistance to the temperature coil.

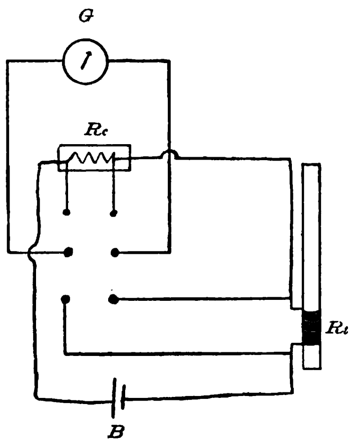

FIG. 69.

This method has been found to be very satisfactory in determining the coeffiicient of the expansion of such bars.*

Another example of the application of this method is found in the determination of the average temperature of the phosphor-bronze suspension wire of the swinging coil described in Lecture IV. This wire was stretched vertically through a distance of two meters, and it formed the suspension of the coil. All attempts to get its temperature by means of mercury thermometers proved very unsatisfactory, but by means of a No. 40 copper wire carried several times the entire length of the suspension tube, very good results were obtained.†

* See the report of Joseph Le Conte upon the coefficient of expansion of a standard meter made by the Societe Genevoise ; reports of the Physical Laboratory of Cornell University, 1891.

† See N. H. Genung's Thesis on the Electro-Chemical Equivalent of Silver, Cornell University Library.

Still another illustration of the application of this method of measuring temperatures is afforded by the experiments of Messrs. Child, Quick and Lanphear* upon the distribution of temperature along a copper bar, one end of which was heated. The object of the experiment was to determine the thermo-conductivity of the bar. For this purpose the distribution of temperatures after the bar had reached its final condition was necessary. A collar consisting of a single layer of very fine insulated copper wire fitting closely around the bar made it possible to measure the temperature at different points throughout the entire length of the latter with great accuracy. In this case, however, the measurements were made by the method of the Wheatstone bridge.

The best form of apparatus for the application of this method is the slide bridge. In Fig. 70, which shows the connections, A B is the slide wire of platinum iridium, P.

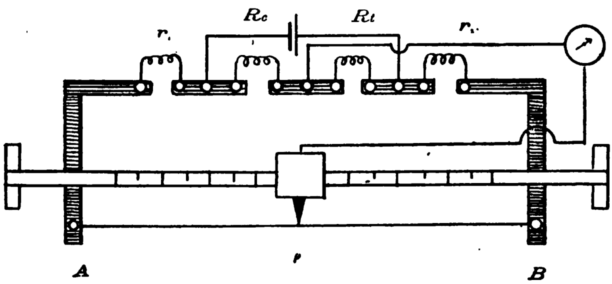

FIG. 70.

the sliding contact, R_t and R_c are the temperature coil and compensated resistance respectively, while r_1 and r_2, taken together with the two parts of the slide wire, are the other arms of the bridge.

With this arrangement of the apparatus the temperature may be conveniently expressed in terms of the position of the sliding pointer P.

The resistance of copper is well adapted for the measurement of temperature, since its changes, through a very wide range, are nearly proportional to the temperature. It is, indeed, only above 200° and below 100° that the change in the coefficient is such as to introduce grave errors.

The coefficients of different wires vary somewhat, however, in absolute value, so that the specimen of which the temperature coil is to be made should always

* Physical Review, Vol. II.

be calibrated. Many samples of modern commercial copper show a coefficient of resistance greatly in excess of Matthiessen's value for the pure metal. Kennelly and Fessenden*, for example, in 1893 found for a copper wire 0.004065, with variations from that value at 27.8° of 0.000058 and at 255° of + 0.000005.

Values lying between .0041 and .0042 are frequently observed in modern practice. Thus Dewar and Fleming found for low temperatures 0.00410; Quick and Lanphear, between — 38.6° C. and + 1.5°C. obtained 0.004147; Cailletet and Bouty's value was higher than any of these, viz.: — 0.00423.

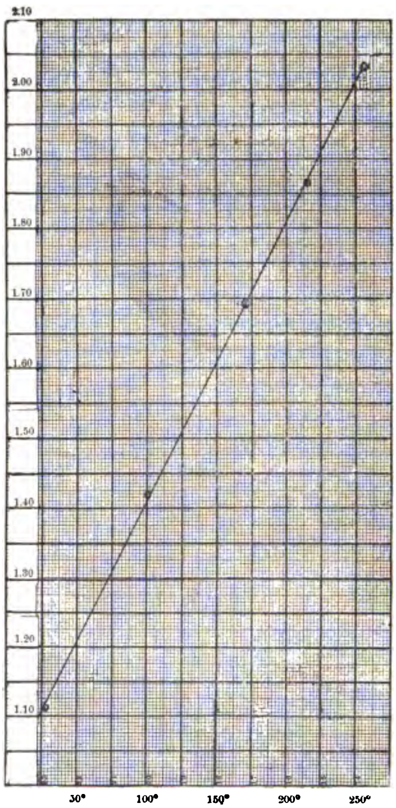

FIG. 71.

The very nearly constant value of the resistance coefficient is shown graphically in Figs. 71 and 72,

*Physical Review, vol. 1.

which are plotted from the results of Kennelly and Fessenden and of Quick and Lanphear respectively.

These curves are plotted to different scales and they apply to different specimens of copper, not of the same quality as to the coefficient of resistance. The methods of calibration also were distinct. Both are straight lines, however, indicating an unvarying coefficient through wide range of temperatures, including the important interval below — 40° C., which cannot be reached with mercury thermometers.

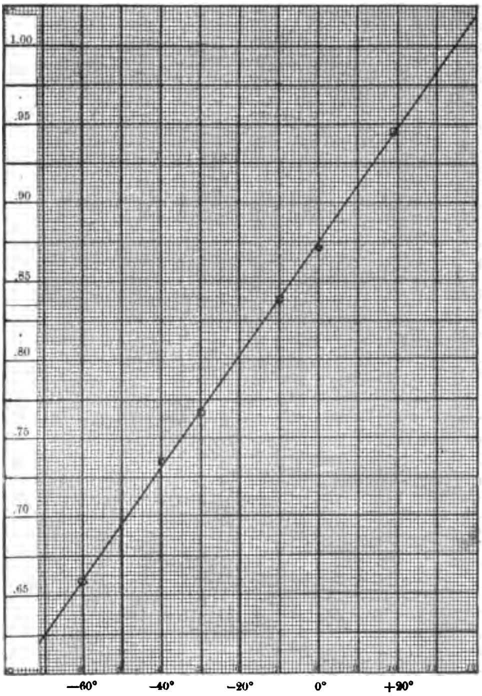

FIG. 72.

The calibration of a temperature coil, should always be made under conditions as nearly as possible, identical with those under which it is to be used. Otherwise the temperature lag, of the coil with reference to the body the temperature of which is to be determined, (or vice versa) will introduce errors which will always be appreciable excepting when temperatures have become strictly stationary, and which may sometimes rise to unsuspected size.

The nature of this error in a typical case, that in which the temperature of a copper bar, was to be measured by means of a collar of fine wire surrounding it, is indicated in Fig. 73 (from determinations by the observers just cited). In this diagram abscissas are temperatures of the bar, as indicated by a thermometer, the bulb of which was immersed in a mercury capsule within the body of the metal, while ordinates are readings on the slide bridge. The bridge gives *relative* readings

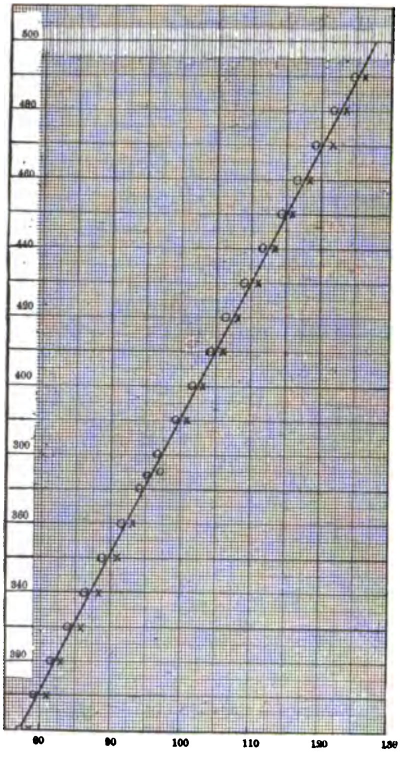

FIG. 73.

of the temperature of the collar. The crosses and circles show the temperatures of the bar at which the bridge readings were the same, for rising and falling temperatures respectively. The curves of heating and cooling to which these apply were very nearly identical and the temperature difference at any point between an observation and the median line, gives the lag (positive or negative). It will be noted that in this case the error of assuming the temperature of the collar to be that of the bar would have been about one degree.

For very high temperatures, copper is not available for electrical thermometry and consequently many attempts have been made to substitute platinum for that metal. The law of resistance for platinum, must however be determined for each specimen and this calibration is a matter of extreme difficulty.

A comparison of the various formulae proposed by Matthiessen, Siemens, and Benoit, for determining temperatures from the resistance of platinum was made some years ago by the writer.* His curves showing the

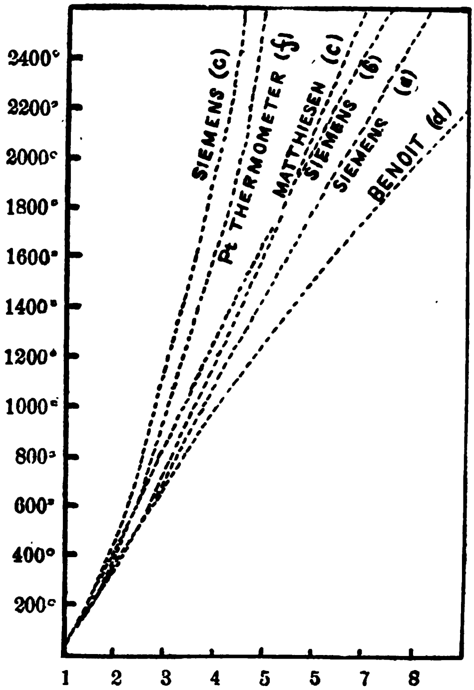

FIG. 74.

divergent character of the results which would be obtained by the application of their formulae, are given in Fig. 74.

Another method, applicable alike to very high and to very low temperatures, is that of the thermo-element of platinum—platinum-iridium. The same caution must be observed, however, with reference to commercial specimens as in the method previously described.

*Am. Journal of Science, vol. 22, p 363.

Platinum wire purchased from leading dealers in this country, and supposed to be pure, gave in the hands of the writer curves of E. M. F. and temperature of the character shown in Fig. 75, one being concave and the other convex to the base line. These wires were combined with the same quality of platinum-iridium in the construction of the thermo elements.

It will be seen that a deflection which corresponds to 600° C. in the one case would be reached, with the other thermo element at 900° C.

Barus, in his exhaustive research upon the measurement of high temperatures, found similar peculiarities

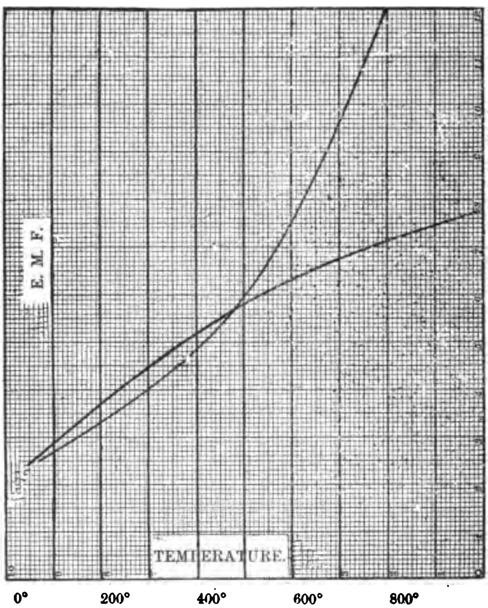

FIG. 75.

in commercial wires, but when he used an element composed of standard materials he obtained a curve which is very nearly straight through a wide range of temperatures.

Excellent results have also been reported with this couple at very low temperatures, but it should be noted that some commercial samples give reversal at a temperature slightly above 0°.

The methods thus far described can be pursued with galvanometers of medium delicacy ; when, however, we come to the exceedingly small temperature differ-

ences with which the student of radiant heat has to deal, instruments of the highest sensitiveness are essential.

Nearly all measurements in this domain are of a relative character ; Knut Angstrom, however, (1893) has described a bolometric method by means of which absolute determinations may be made, and the results expressed in *gram-calories per second per c m*^{2}.

The principle of Angstrom's method, as described in his paper,* is briefly as follows : "Given two thin strips of metal, A and B (Fig. 76), which are as nearly as possible identical. The sides of these, which are exposed to the source of heat, are blackened, and the strips are arranged in such a way that it is possible to determine accurately when they are of the same temperature. These strips are so placed that a current of any desired strength can be sent through them. If one of them, for example, A, is exposed to the source of heat, while

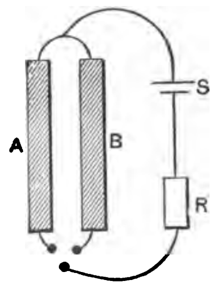

FIG. 76.

B is protected by a screen, we may restore the balance of temperature which has been disturbed by the absorption of heat on the part of A, by sending a current of proper intensity through B. When the temperatures are the same, then the amounts of energy which A and B have received are equal to one another. Let l and b be the length and width of the strips, r the resistance of the same, and i the current. Then since the heat absorbed by A is the equivalent of that produced by the electric current in b, we may write

$$q\,l\,b = \frac{r\,i^2}{4.2}, \text{ or } q = \frac{r\,i^2}{4.2\,l\,b},$$

where q is the radiant energy received by a unit of surface. In order to counteract the inequality of the strips, they are interchangeable, B being illuminated and the current sent through A.

*Angstrom ; Trans. Royal Soc. of Sc. Upsala, 1893 ; also *Physical Review*, vol. 1, p. 365.

"In the practical application of this principle of compensation, one may follow various methods. The equality of temperature may be determined in a variety of ways. If thermo-elements are used for this purpose, one needs a sensitive galvanoscope, and the measurement of the current can be made upon an instrument of ordinary delicacy. It is possible, however, to carry on the investigation without any difficulty, using only one galvanometer, a plan which I pursued in the case of the first apparatus which I constructed. Fig. 77 gives a diagram of the connections.

"The metallic strips, A and B, cut simultaneously from two thin sheets of platinum laid one upon another, are 0.154 cm. wide and 1.80 cm. in length. They are blackened in the usual way upon the side exposed to radiation, and are mounted side by side in a frame of ebonite.

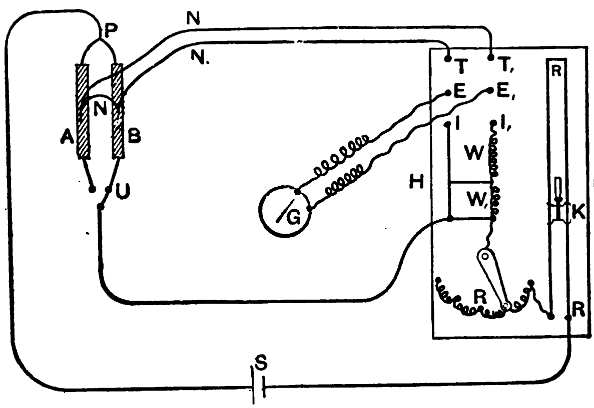

FIG. 77.

This frame is inserted in a tube. Upon the back of the strips are laid two exceedingly thin leaves of mica, upon which very minute junctions of copper and German silver, are inserted. To hold the mica and the thermojunctions together, I used marine glue in as thin a layer as possible.

"One of the strips is subjected to radiation; the circuit is then closed through the other one; the thermo-element is brought into circuit with the galvanometer by means of the commutator; the sliding contact is adjusted until the galvanometer stands at zero; the commutator is then reversed, and the strength of the current used in heating the strip is determined. The switch is then reversed, the shutter is placed in front of the other strip, and the setting and current measure-

ment are repeated. There are two other ways in which one can use this apparatus for the measurement of radiation, viz :—

First Variation of the Method,—One of the strips, for example, A, is exposed to radiation, while the other is screened. One notes the deflection of the galvanometer, which becomes constant in about fifteen seconds. A is then also screened, and by means of the current is brought to the same temperature to which radiation had previously brought it. The strength of the current producing this rise of temperature is then measured. The advantage of this arrangement is that the same strip is warmed by means of radiation and then through the agency of the current, so that it is not necessary to be so painstaking as regards the identity of the two strips.

Second Variation of the Method.—One of the strips, for example, A, is exposed to radiation, the other, B, is screened ; the deflection of the galvanometer is observed after the thermo-current has become constant. The quantity of heat which A has received is calculated from Newton's law of cooling.

$$Q = b\,l\,q = k\,\theta,$$

where θ is the difference of temperature indicated by the galvanometer, and k is the constant of cooling of the strip. If we now send a current through A, without making any change of conditions, the difference of temperature will become greater still. This will be indicated by the resulting deflection θ_1. The strength of the current producing this additional heating effect may be measured in the manner already described."

Galvanometer, suitable for investigations are easily constructed, following the general principles of design given in Lecture VI.

In undertaking any extended bolometric work, or other research of great delicacy, the galvanometer should be especially constructed for the particular purpose which it must serve. The question of the period of oscillation is sometimes an important one ; on account of local magnetic disturbances. The proper adaptation of the resistance to the circuit in which the instrument is to be used is of especial significance.

Very often, the experimenter with the bolometer is compelled to use his galvanometer under conditions of the highest attainable delicacy. Then, if he is to obtain readings which possess value, his patience and skill will be taxed to the utmost. For the difficulties which arise in such researches no general prescription can be written. The vagaries of the needle under the

combined influences of diurnal magnetic drift, local magnetic disturbance, thermo-electric differences, and the obscure thermal fluctuations to which the bolometer and its accessories are subject must be studied as they arise, and overcome. These causes can be detected, and, after due experience, ingenuity and tireless patience will bring their effects under control; but more remote sources of disturbance are perpetually at work against the bolometrist: A moderate gale of wind, an auroral display, almost too faint to be visible, even a storm in the solar atmosphere, will drive him from his seat at the reading telescope in despair.

These are difficulties to be surmounted only by him who can wait; he it is, alone, who may enter the realms of research which lie along the very boundary line of human attainment; in his hands alone do we see the highest performance of that remarkable instrument, the galvanometer.

FINIS.